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Design and analysis of mechanisms for decentralized joint replenishment

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ABSTRACT

We consider jointly replenishing multiple firms that operate under an EOQ like environment in a decentralized, non-cooperative setting. Each firm's demand rate and inventory holding cost rate are private information. We are interested in finding a mechanism that would determine the joint replenishment frequency and allocate the joint ordering costs to these firms based on their reported stand-alone replenishment frequencies (if they were to order independently). We first provide an impossibility result showing that there is no direct mechanism that simultaneously achieves efficiency, incentive compatibility, individual rationality and budget-balance. We then propose a general, two-parameter mechanism in which one parameter is used to determine the joint replenishment frequency, another is used to allocate the order costs based on firms' reports. We show that efficiency cannot be achieved in this two-parameter mechanism unless the parameter governing the cost allocation is zero. When the two parameters are same (a single parameter mechanism), we find the equilibrium share levels and corresponding total cost. We finally investigate the effect of this parameter on equilibrium behavior. We show that properly adjusting this parameter leads to mechanisms that are better than other mechanisms suggested earlier in the literature in terms of fairness and efficiency.

1. Introduction

The classical Economic Order Quantity (EOQ) model is a well-known and studied model in inventory management literature. The core of this model is the trade-off between inventory holding costs and setup costs associated with production, transportation or procurement. In the simplest form of the model, a firm faces deterministic demand with a constant rate, pays a setup cost for each replenishment order and incurs inventory holding costs for each unit of inventory it carries per unit of time. Minimizing setup and inventory holding costs gives the famous formula for the optimal order quantity. Since the first study (Harris, 1913), there has been a vast amount of literature on EOQ model, its extensions and the more general lot sizing problem. The interested reader is referred to Jans and Degraeve (2008) for a recent review.

A major cost saving opportunity in this setting is to consolidate orders for different items (or locations). By carefully coordinating the replenishment of multiple items that may incur a joint setup, one can exploit the economies of scale of ordering jointly and reduce setup costs, inventories or both. This problem is known as Joint Replenishment Problem (JRP) and there is a growing literature in this area since 1960s. See Khouja and Goyal (2008) and Aksoy and Erengüç (1988) for two important reviews of research on this problem. The basic assumption in this literature is that the items or locations that are replenished jointly are also controlled centrally. However, this may not be always true. With intense and increasing pressure to reduce costs, independent, and sometimes competing firms may also be interested in jointly replenishing their inventories. For example, recently, BMW started an auto-parts purchasing partnership with one of its main competitors, Daimler, to procure more than 10 parts together and looking for ways to expand this partnership. BMW hoped to generate cost savings of around 100 million Euros annually through this venture (Gilbert, 2010). The advent of the Internet and B2B exchanges made collaborative purchasing and replenishment easier than ever and led to large scale and successful purchasing consortiums or groups. A recent review article states that collaboration is one of the most important trends and research opportunities in supply chain management (Speranza, 2016).

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1.1. Related work

Decentralized joint replenishment has attracted attention in literature only recently and studies until now investigate how the total savings (or total costs) should be allocated among participants using cooperative game theory. Meca, Timmer, Garcia-Jurado, and Borm (2004) propose a coordination scheme where the players only share their independent order frequencies prior to joint replenishment. Their allocation mechanism distributes the total setup cost among the players in proportion to the square of their order frequencies. They show that this allocation is in the core of the game. Fiestras-Janeiro, García-Jurado, Meca, and Mosquera (2015) study the case where the warehouse space for each player is limited, but the inventory holding costs are negligible. Timmer, Chessa, and Boucherie (2013) extend the work of Meca et al. (2004) for stochastic demand and suggest two coordination strategies.

When minor setup costs associated with each ordered item are also present, it may not be optimal to order every item with every replenishment. In fact, the structure of optimal policy is not known. For this problem, Hartman and Dror (2007) show that the game with a specific group of items has a core, whenever these items need to be ordered together on the same schedule to minimize total costs. Anily and Haviv (2007) focus on near optimal power-of-two policies for this problem, and show the existence and example of a core allocation. Zhang (2009) generalizes these results for the case of a sub-modular joint setup cost function and orders passing through a warehouse that may carry inventory. Minner (2007) uses bargaining models to study the collaboration between firms in a similar joint replenishment setting. For a recent review of research that uses cooperative game theory in inventory theory, see Fiestras-Janeiro, Garcia-Jurado, Meca, and Mosquera (2011).

In this paper, we follow a non-cooperative approach for the joint replenishment problem. Bauso, Giarre, and Presenti (2008) consider a periodic inventory model where each firm needs to determine the order quantities in each period to satisfy its demand. The demand in each period is different but known in advance. The fixed order cost is shared among multiple firms that order in the same period. They show the existence of pure strategy Nash equilibria and propose a consensus protocol that reaches to one of these equilibria. In Meca, Garcia-Jurado, and Borm (2003), each firm reports an order frequency (that may be different from its true order frequency) and the joint order frequency is determined to minimize the total joint costs based on these reports. Each firm incurs holding cost individually and pays a share of the joint replenishment cost in proportion to the squares of reported order frequencies. It is shown that this rule entails significant misreporting and inefficiency. It is shown that the game has multiple equilibria, in one of which none of the firms participate in joint replenishment. If the firms are sufficiently homogeneous, there also exists a (unique) “constructive equilibrium”, i.e., an equilibrium in which all firms participate in joint replenishment.

Körpeoğlu, Şen, and Güler (2012) follow a more direct approach using a two stage game. They assume that there is an intermediary that coordinates the replenishment activity. In Stage 1, each firm decides whether to participate in joint replenishment by agreeing to pay a minimum contribution or to replenish independently. In Stage 2, each participating firm submits a contribution to the intermediary. Then, the intermediary determines the minimum cycle time that can be financed with these contributions. It is shown that all firms participate in equilibrium and only those firms with the highest adjusted demand rates pay more than the minimum contribution. Körpeoğlu, Şen, and Güler (2013) study the private information version of the game in Körpeoğlu et al. (2012). It is shown that the privacy of information eliminates free-riding but contributions are not as high yielding higher aggregate costs.

1.2. Contributions

In this paper, we study the mechanism design problem for the joint replenishment of decentralized firms which have private information about their demand rates and inventory holding cost rates. We first study a direct mechanism where each firm reports its independent frequency and a joint replenishment frequency and the allocation of the joint order costs between the firms are decided based on these reports. We show that a direct mechanism which satisfies the efficiency, incentive compatibility and individual rationality constraints cannot satisfy the budget-balance constraint, i.e., a truth telling direct mechanism cannot finance the joint replenishment for efficient cycle times. Next, we generalize the mechanism suggested by Meca et al. (2003). While the mechanism in Meca et al. (2003) determines the joint order frequency and the order cost allocation both based on the *squares* of the reported stand-alone order frequencies, we use a general formulation in which two separate parameters govern these decisions. For this two-parameter mechanism, we show that the joint frequency is always lower than the efficient frequency unless the order cost is allocated uniformly. We then study the one-parameter mechanism, where these two parameters are equal to each other. We find the conditions necessary for a constructive equilibrium and characterize this equilibrium. We also provide necessary conditions for convexity at the equilibrium point. We analyze the comparative statics of the one-parameter model and show that using smaller values of this single parameter leads to better mechanisms in terms of fairness and efficiency.

2. The model and preliminaries

We consider a stylized EOQ environment with a set of firms $N = \{1, \ldots, n\}$. Demand rate for firm i is constant and deterministic at β_i per unit of time. Inventory holding cost per unit time for firm i is γ_i per unit. We denote the adjusted demand rate of firm i as $\alpha_i = \gamma_i\beta_i$. We assume that adjusted demand rates are strictly positive, $\alpha_i > 0$ for all $i \in N$ to rule out trivial replenishment environments where either the demand rate or the holding cost rate is zero. Major ordering cost is fixed at κ per order regardless of order size. Minor ordering costs (ordering costs associated with firms included in an order) are assumed to be zero. We assume that the outside supplier that replenishes the orders has infinite capacity. The firms aim to minimize their long-run *average* costs over time and backorders are not allowed.

In any setting, the objective is to minimize the total cost rate, denoted by C, i.e., the sum of replenishment cost rate (R) and holding cost rate (H): $C = R + H$. The decision variable can be taken as order cycle time, t, or order frequency, $f = 1/t$ (number of orders per time unit). We take frequency as the decision variable in the sequel.

Vectors are denoted by lower-case letters in bold typeface. For an endogenous variable X, by X_M^a we refer to the value of X when the set of firms is M and replenishment operations are governed by $a \in \{c, d, 2p, 1p\}$, where c stands for centralized, d stands for decentralized (or independent) replenishment, $2p$ stands for two-parameter mechanism and $1p$ stands for the single-parameter mechanism. For instance, C_M^c is the total cost of the firms in M when replenishment is centralized. When the set M is a singleton, e.g., $M = \{i\}$, we use X_i^a instead of $X_{\{i\}}^a$. Exceptions to this notation are used for f_i, the optimal frequency of the decentralized replenishment for firm i and for f_*, the optimal frequency of centralized replenishment.