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Decentralized matching markets with endogenous salaries

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Abstract

In a Shapley–Shubik assignment problem with a supermodular output matrix, we consider games in which each firm makes a take-it-or-leave-it salary offer to one applicant, and a match is made only when the offer is accepted by her. We consider both one-shot and multistage games. In either game, we show that there can be many equilibrium salary vectors which are higher or lower than the minimum competitive salary vector. If we exclude artificial equilibria, applicants' equilibrium salary vectors are bounded above by the minimal competitive salary vector, while firms' equilibrium payoff vectors are bounded below by the payoff vector under the minimum competitive salary vector. This suggests that adopting the minimum competitive salary vector as the equilibrium outcome in decentralized markets does not have a strong justification.
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1. Introduction

As is described in Roth (1984) and Roth and Sotomayor (1990) the centralized matching procedure in the US medical resident market—the National Residency Matching Program (NRMP)—has been a tremendous success. NRMP uses the deferred-acceptance matching algorithm developed in Gale and Shapley (1962) with a salary vector chosen by hospitals prior to the matching procedure in order to assign senior medical students to residency programs at participating hospitals. The introduction of NRMP helped reduce the early contract craze and

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last-minute congestion that resulted under the decentralized system, and the NRMP participation rate by both students and hospitals has been very high.

In 2002, however, a lawsuit against teaching hospitals and NRMP was filed alleging that the system violated federal antitrust law (in that, for example, it restrained competition). This lawsuit could have had a significant impact on the medical market: one possible consequence was abandonment of the NRMP and other medical matching programs.[1]

Recently, Bulow and Levin (2006) set up an interesting matching model that can compare the centralized matching mechanism that has characteristics of NRMP with a decentralized market. Bulow and Levin (2006) employ a simplified version of the assignment model in Shapley and Shubik (1971), and consider a two-stage game. In the first stage, hospitals simultaneously decide salaries, and in the second stage the Gale–Shapley deferred-acceptance matching algorithm takes place, using the preferences of hospitals and residents generated by the price vector determined in the first stage. Thus, their game mimics the matching program of the NRMP. Bulow and Levin (2006) characterize a mixed-strategy equilibrium of the game, and compare the expected equilibrium salary of each resident in the game with her *minimum competitive salary*. The minimum competitive salary (vector) is the lowest market equilibrium salary vector under which the surplus-maximizing assignment of hospitals and residents is stable (Shapley and Shubik, 1971). The main finding of their paper is that under the centralized system, salaries and applicants' payoffs are more suppressed than they are under the decentralized system.

Although the result of Bulow and Levin (2006) appears to suggest the benefits of a decentralized market,[2] the notion of the minimum competitive salary does not really fit with an equilibrium outcome of a decentralized resident-hospital matching market. Since each resident is a heterogeneous commodity, the minimum competitive salary can be attained as a result of a Vickrey auction in the *centralized multi-object auction* market. However, this mechanism clearly also requires a centralized auction market. It is not clear what kind of decentralized salary vector emerges under a decentralized market given the typical bilateral job offers in the resident-hospital market.

In this paper, we analyze equilibria of noncooperative games that describe decentralized markets using the Shapley–Shubik assignment model. Following Bulow and Levin (2006), we assume that the output matrix is supermodular (slightly more general than the one in Bulow and Levin), and we consider games in which each firm chooses an applicant and makes a take-it-or-leave-it salary offer and a match is made only when the offer is accepted by the applicant.[3] We consider both one-shot and multistage games. Although these games are too simplistic to describe to be taken to real-world market institutions, they can be regarded as a first step in an attempt to analyze the equilibrium salary vectors in a decentralized labor market for entry-level professionals such as the one for medical interns.

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[1] This lawsuit was dismissed in 2004 following the president's signing a rider law (the Pension Funding Equity Act of 2004) exempting NRMP from antitrust lawsuits.

[2] Bulow and Levin (2006) caution that their result does not directly indicate that NRMP suppresses the wages of medical residents, since NRMP can use enormous amounts of information nationally whereas a decentralized system tends to match agents locally.

[3] We adopt a setup in which firms make only a limited number of offers (constrained by the number of slots) to applicants, following the literature on timing of transactions and congestion in market clearing (Roth and Xing, 1994; Niederle and Roth, 2007; and Niederle et al., 2006). As examples of those markets, Roth and Xing (1994) list many entry-level professional labor markets including the US medical intern markets mentioned in Roth (1984) and Bulow and Levin (2006), and the adopted setup applies to these markets. The "take-it-or-leave-it" offer assumption is certainly for simplification. This assumption is also commonly made in the literature.