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Dominance solvable English matching auctions

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Abstract

It is well-known that the competitive allocation in a one-to-one matching market can be constructed with the Gale–Shapley algorithm. In this algorithm the buyers use a simple Walrasian bidding scheme to increase the market price until the excess demand vanishes. It is therefore natural to ask under which conditions rational buyers should use this bidding scheme in an English multi-object auction. This paper presents two auction games in which an iterated elimination of dominated strategies justifies the expected outcome. © 2001 Elsevier Science B.V. All rights reserved.

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1. Introduction

Matching as a problem of strategic interaction was introduced in a seminal paper by Gale and Shapley (1962). They considered a model in which the members of two distinct groups of agents (the men and the women) can improve upon their status quo if they are matched to a member of the opposite group. Every agent has a strict ranking of all the opponents (including the option of staying alone) so that an allocation is determined by a one-to-one matching between two subsets of the two groups. Gale and Shapley called an allocation stable if there are no two agents who could both improve their situation if they were matched to each other instead. They proposed to construct a solution with the following repetition of offers and rejections: each man proposes a

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match to his most preferred partner; each woman keeps only the proposal which she likes most, and the rejected men become free to turn to their next best partner. After finitely many iterations this algorithm converges to the unique men-optimal stable allocation which is preferred by all the men to all other stable solutions.

The Gale–Shapley algorithm is not only interesting as a computation device, it is also useful to describe economic reality. Thus it was used recently by Blum et al. (1997) to explain vacancy chains which are caused by job openings in certain high skill labor markets. The descriptive appeal is even stronger if one looks at Crawford and Knoer's (1981) extension of the Gale–Shapley algorithm to allocation problems in which the matched agents have a natural conflict of interest over the price of the traded good. Instead of turning to next best partners the potential buyers now submit next best price offers so that the Gale–Shapley algorithm computes a history of myopically optimal bids in an English auction in which several (different) objects are sold simultaneously. Moreover, the set of stable solutions in this matching market coincides with the set of competitive equilibria[1], while each iteration of the generalized Gale–Shapley algorithm can also be interpreted as an increase of the prices for the over-demanded objects. Thus Crawford and Knoer provide a market model in which the Walrasian tâtonnement process has the desired convergence properties and describes a natural myopic bidding behavior in a mechanism which can be implemented as an English multi-object auction.[2] But in spite of the conceptual importance of the Walrasian tâtonnement, the descriptive appeal of the Gale–Shapley algorithm, and the enormous impact which this algorithm had (and still has) on the literature on two-sided matching markets no one has analyzed the strategic structure of such English matching auctions.

So far the studies of the strategic aspects of the matching problem have been concentrated on revelation games. The papers by Dubins and Freedman (1981), Roth (1982) and Demange and Gale (1985) prove that it is a dominant strategy for the buyers to reveal their true preference order to a central agency who constructs the buyer-optimal stable solution (for instance with the Gale–Shapley algorithm). This property is very important as it generalizes Vickrey's (1961) famous dominance result to the matching model, but it is only of limited interest for the analysis of real life mechanisms. Already Vickrey introduced his second price auction not to describe observable auctions but as 'logically equivalent'[3] to the English auction which he wanted to study. The Gale–Shapley algorithm can describe some existing organized markets if there are no transfer payments involved[4], but it becomes an artificial planning device if prices have to be determined together with the matching. It is therefore important to know to what extent the dominance solvability of the (generalized) second price auctions also holds for the corresponding English matching auction.

[1]For a very thorough introduction to matching problems and the related literature see Roth and Sotomayor (1990).

[2]This observation was formalized for two English multi-object auctions by Demange et al. (1986).

[3]Vickrey (1961, p.20). Similar statements are explicit in the surveys by McAfee and McMillan (1986, p.708), Milgrom (1985, p.273) or Kagel (1995).

[4]A revelation mechanism for the matching of interns to hospitals is described by Roth and Sotomayor (1990). For a discussion of such institutions with more or less formal rules see Roth and Xing (1995).