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A fuzzy GARCH model applied to stock market scenario using a genetic algorithm

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ABSTRACT

In this paper, we derive a new application of fuzzy systems designed for a generalized autoregression conditional heteroscedasticity (GARCH) model. In general, stock market performance is time-varying and nonlinear, and exhibits properties of clustering. The latter means simply that certain large changes tend to follow other large changes, and in general small changes tend to follow other small changes. This paper shows results from using the method of functional fuzzy systems to analyze the clustering in the case of a GARCH model.

The optimal parameters of the fuzzy membership functions and GARCH model are extracted using a genetic algorithm (GA). The GA method aims to achieve a global optimal solution with a fast convergence rate for this fuzzy GARCH model estimation problem. From the simulation results, we have determined that the performance is significantly improved if the leverage effect of clustering is considered in the GARCH model. The simulations use stock market data from the Taiwan weighted index (Taiwan) and the NASDAQ composite index (NASDAQ) to illustrate the performance of the proposed method.

1. Introduction

In analyzing time-dependent data, it is often the case that the conditional variances are not consistent with the assumption of homogeneity that is commonly associated with traditional econometrics models, especially those which treat financial data (Arciniegas & Rueda, 2008; Chan, 2002; Fama, 1965; Tsay, 2002). Mandelbrot (1963) discovered that conditional variance plays a role is in the phenomenon of volatility clustering. Volatility clustering means that large changes tend to follow other large changes, and small changes tend to follow small changes. Because of this phenomenon, Mandelbrot thought that the variance might change over time, that is, it would not be constant or homogeneous. Therefore, Engle (1982) proposed the autoregression conditional heteroscedasticity (ARCH) construct. Engle believed that conditional variances led to assumption of homogeneity. However, this approach proved impractical. He subsequently adopted a model in which conditional variances in time-dependent data were subject to influences from previous unexpected factors. Furthermore, he assumed that the conditional variances were functions of error terms, allowing them to change over time. He proposed that the ARCH Model could solve the biases and therefore address traditional econometrics models.

Building on Engle's ARCH(q) model, Bollerslev (1986) made use of the Autoregressive Moving Average (ARMA) model to introduce the GARCH model. The GARCH model uses prior conditional variances to estimate the degree of transmission of volatility; it is characterized by a fat tail and excess kurtosis. Its ability to explain the transmission of volatility is a key advantage of this approach. For these reasons, the GARCH model is frequently used to explore the returns and transmissions of volatility in time-dependent financial data sets. However, financial assets are easily impacted by both positive and negative information, and the impacted are existed the asymmetric. The GARCH model does not recognize transmissions of volatility that derive from the input of positive or negative information. Therefore, this model is not appropriate when the market is asymmetric.

To address this issue, other researchers introduced various asymmetric GARCH models. The GJR GARCH model was proposed by Glosten, Jagannathan, and Runkle (1993), while the exponential GARCH (EGARCH) was put forward by Nelson (1991). These models suggest that the negative relation between volatility and stock prices can be understood by the fact that an increase in unexpected volatility will increase the expected future volatility (assuming persistence). Some of these effects can be captured by modifications of linear models, but others demand nonlinear approaches. Unfortunately, because of their complexity, nonlinear models are in very limited use today.

During the past two decades, fuzzy systems including Sugeno systems (Takagi & Sugeno, 1985) and Mamdani systems are capable of approximating a wide range of functions. The Sugeno or functional fuzzy systems have recently found extensive application in a wide variety of industrial systems and consumer products and have attracted the attention of many control researchers due to their unique model-independent approach (Lee & Chen, 2008; Liu

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& Wang, 2007; Lee & Shin, 2003; Savran, 2007; Treesatayapun & Uatrongjit, 2005). This paper proposes a new class of GARCH models that are based on functional fuzzy systems. Methods of fuzzy modeling are promising techniques for describing complex dynamic systems. Combining the ease of implementation and convenience of linear models with an ability to capture complex system correlations, we propose that fuzzy models could also be a judicious choice for analyzing dynamic processes that feature time-dependent variances. In this paper, we combine GARCH models and functional fuzzy systems and we apply these new models to real-world financial markets using GA. The process of optimizing functional fuzzy systems and GARCH model parameters is highly complex and nonlinear. A GA-based parameter estimation algorithm is proposed to derive the optimal solution for the fuzzy GARCH model.

GA is a method for optimizing machine learning algorithms inspired by the processes of natural selection and genetic evolution (Goldberg, 1989; Crefensteet, 1986; Holland, 1962). GA applies operators to a population of binary strings that encode the parameter space. A parallel global search technique emulates natural genetic operators such as reproduction, crossover, and mutation. At each generation, the algorithm explores different areas of the parameter space and then directs the search to the region where there is a high probability of finding improved performance. Because GA simultaneously evaluates many points in a parameter space, it is more likely to ultimately converge on the global solution. In particular, there is no requirement that the search space is differentiable or continuous, and the algorithm can iterate several times on each data point. Accordingly, it is a very suitable approach for time-varying nonlinear functions (Zhou & Khotanzad, 2007).

To use the genetic algorithm in the problem of fuzzy GARCH model parameter estimation, the relevant variables are first coded into a binary string called a chromosome. In each generation, three basic genetic operators (reproduction, crossover, and mutation) are performed to generate a new population with a constant population size. The chromosomes that remain after the population is reduced by the principle of survival of the fittest produce a better population candidate solution. The convergence of the proposed GA estimation scheme can be guaranteed via the theorem of the schema discussed in Holland (1975) and in Toroslu (2007). The estimation parameter that is obtained by the proposed estimation scheme ultimately converges to the optimal or near-optimal solution.

The rest of this paper is organized as follows. The next section describes the problem. Thereafter, section three presents the details of the fuzzy GARCH model. The fourth section discusses the proposed GA-based optimization of the fuzzy GARCH system. Experimental results that illustrate the effectiveness of the proposed method are provided in the fifth section. Conclusions are in the final section of the paper.

2. Problem description

Consider a ARCH(q) model that is defined as (Engle, 1982)

$$\begin{aligned} y(t) &= a(t) \\ a(t) &= \sqrt{h(t)}\varepsilon(t) \\ h(t) &= \alpha_0 + \sum_{i=1}^{q} \alpha_i a^2(t-i) \end{aligned} \tag{1}$$

where $y(t)$ is a random variable representing certain stock market data, $\varepsilon(t)$ is a zero mean and unit variance white noise random process, $h(t)$ is the conditional variance of $\varepsilon(t), t$ is the time index, and α_0, α_i are nonnegative; $\alpha_0 > 0, \alpha_i \geqslant 0$.

Bollerslev (1986) modified the conditional variance term in the ARCH(q) model, by assuming that the conditional variances are influenced not only by the squared error terms, but also by previous conditional variances. He incorporated previous conditional variances into the process for estimating transmission of volatility. The result was his proposed GARCH(p, q) model. The model is defined as

$$\begin{aligned} y(t) &= a(t) \\ a(t) &= \sqrt{h(t)}\varepsilon(t) \\ h(t) &= \alpha_0 + \sum_{i=1}^{q} \alpha_i a^2(t-i) + \sum_{j=1}^{p} \beta_j h(t-j) \end{aligned} \tag{2}$$

where $\alpha_0, \alpha_i,$ and β_j are unknown parameters that must be estimated. Without loss of generality, we assume

$$\begin{aligned} &\alpha_0 > 0, \alpha_i \geqslant 0; \quad i = 1, 2, \ldots, q;\ q > 0 \\ &\beta_j \geqslant 0; \quad j = 1, 2, \ldots, p;\ p > 0 \\ &\sum_{i=1}^{q} \alpha_i + \sum_{j=1}^{p} \beta_j < 1 \end{aligned} \tag{3}$$

In generally, the GARCH model can produce relatively exact results for different financial data sets. Accordingly, the conventional GARCH model has long been considered the most popular instrument for analyzing time-dependent data that features time-varying conditional variance. However, the model does not perform well when the market features clustering behavior. Ignoring this fact can lead to poor prognostic characteristics. Consequently, we adopt functional fuzzy systems to propose a new fuzzy GARCH model.

3. Fuzzy GARCH model

Fuzzy logic systems are universal approximations that can uniformly estimate nonlinear continuous functions with arbitrary accuracy. The functional fuzzy model is a piecewise interpolation of several models that operates by means of membership functions. The fuzzy model is described by IF-THEN rules and will be employed here ensure the GARCH model can appropriately deal with the cluster problem. The l-th rule of the functional fuzzy system for GARCH is described by
$Rule^{(l)}$: IF $x_1(t)$ is F_{l1} and $\cdots$ and $x_n(t)$ is F_{ln}, THEN

$$\begin{aligned} h(t) &= \alpha_0^l + \sum_{i=1}^{q} \alpha_i^l y^2(t-i) + \sum_{j=1}^{p} \beta_j^l h(t-j) \\ y(t) &= \sqrt{h(t)}\varepsilon(t), \ \text{for } l = 1, 2, \ldots, L \end{aligned} \tag{4}$$

where $y(t)$ is output of system, F_{lj} for $j = 1, \ldots, n$ is the fuzzy set, L is the number of IF-THEN rules, and $x_1(t), x_2(t), \ldots, x_n(t)$ are the premise variables. It is challenging to provide universal recommendations for choosing the set of explanatory variables for a fuzzy rule system (4). In this paper we used what is arguable a "natural" definition for them, namely the previous values of the time series

$$x_i(t) = y(t-i), \ \text{for } i = 1, 2, \ldots, n \tag{5}$$

As seen in (5), the fuzzy model successfully captures the leveraging effects, which is resulted from the sign of the previous time series values.

The fuzzy system is inferred can be written as follows

$$\begin{aligned} h(t) &= \frac{\sum_{l=1}^{L} u_l(x(t)) \left[\alpha_0^l + \sum_{i=1}^{q} \alpha_i^l y^2(t-i) + \sum_{j=1}^{p} \beta_j^l h(t-j)\right]}{\sum_{l=1}^{L} u_l(x(t))} \\ &= \sum_{l=1}^{L} g_l(x(t)) \left[\alpha_0^l + \sum_{i=1}^{q} \alpha_i^l y^2(t-i) + \sum_{j=1}^{p} \beta_j^l h(t-j)\right] \end{aligned}$$