Physica A 391 (2012) 209–216

Contents lists available at SciVerse ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

The level crossing and inverse statistic analysis of German stock market index (DAX) and daily oil price time series

F. Shayeganfar [a,*], M. Hölling [b], J. Peinke [b], M. Reza Rahimi Tabar [a,c]

[a] Department of Physics, Sharif University of Technology, P.O.Box 11365-9161, Tehran 11365, Iran

[b] Carl von Ossietzky University, Institute of Physics, D-26111 Oldenburg, Germany

[c] Fachbereich Physik, Universität Osnabrück, Barbarastraße 7, 49076 Osnabrück, Germany

ARTICLE INFO

Article history:
Received 1 January 2011
Received in revised form 18 June 2011
Available online 12 August 2011

Keywords:
Level crossing
Probability
Inverse statistics
Waiting time

ABSTRACT

The level crossing and inverse statistics analysis of DAX and oil price time series are given. We determine the average frequency of positive-slope crossings, ν_α^+, where $T_\alpha = 1/\nu_\alpha^+$ is the average waiting time for observing the level α again. We estimate the probability $P(K, \alpha)$, which provides us the probability of observing K times of the level α with positive slope, in time scale T_α. For analyzed time series, we found that maximum K is about $\approx$6. We show that by using the level crossing analysis one can estimate how the DAX and oil time series will develop. We carry out the same analysis for the increments of DAX and oil price log-returns (which is known as inverse statistics), and provide the distribution of waiting times to observe some level for the increments.

1. Introduction

Stochastic processes occur in many natural and man-made phenomena, ranging from various indicators of economic activities in the stock market, velocity fluctuations in turbulent flows and heartbeat dynamics, etc. [1]. The level crossing analysis of stochastic processes has been introduced by (Rice, 1944, 1945) [2–26], and used to describe the turbulence [16], rough surfaces [27], stock markets [28], Burgers turbulence and Kardar–Parisi–Zhang equation [29,30]. The level crossing analysis of the data set has the advantage that it gives important global properties of the time series and do not need the scaling feature. The almost of the methods in time series analysis are using the scaling features of time series, and their applications are restricted to the time series with scaling properties. Our goal with the level crossing analysis is to characterize the statistical properties of the data set with the hope to better understand the underlying stochastic dynamics and provide a possible tool to estimate its dynamics. The level crossing and inverse statistics analysis can be viewed as the complementary method to the other well-known methods such as, detrended fluctuation analysis (DFA) [31], detrended moving average (DMA) [32], wavelet transform modulus maxima (WTMM) [33], rescaled range analysis (R/S) [34], scaled windowed variance (SWV) [35], Langevin dynamics [36], detrended cross-correlation analysis [37], multifactor analysis of multiscaling [38], etc.

We start with formalism of the level crossing analysis. Consider a time series of length n given by $x(t_1), x(t_2), \ldots, x(t_n)$ (here $x(t_i)$ is the log-return of DAX and oil prices). The log-return $x(t_i)$ is defined as $x(t_i) = \ln(y_i/y_{i-1})$, where y_i is the price at time t_i. Let N_α^+ denote the averaged number of *positive* slope crossing of $x(t) = \alpha$ in time scale $T = n\Delta t$ with $\Delta t = 1$ (we set also the average $\langle x \rangle$ to be zero). The averaged N_α^+ can be written as $N_\alpha^+(T) = \nu_\alpha^+ T$, where ν_α^+ is the average frequency of positive slope crossing of the level α. The positive level crossing has specific importance that it gives the next average time

* Corresponding author.
E-mail address: fshayeganfar@gmail.com (F. Shayeganfar).

0378-4371/$ – see front matter © 2011 Elsevier B.V. All rights reserved.
doi:10.1016/j.physa.2011.07.037

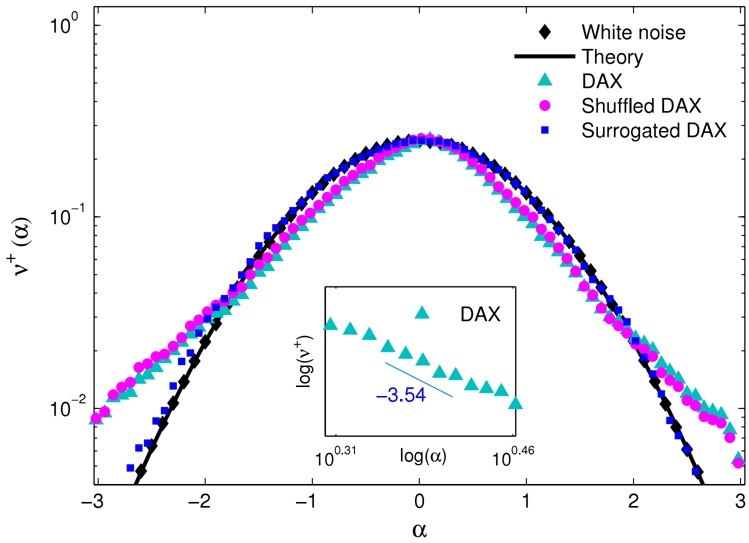

Fig. 1. The level crossing analysis of the DAX log-returns (original, shuffled and surrogated) and uncorrelated Gaussian time series. Inset: the log–log plot of level crossing frequency versus level α for DAX log-return time series. The Gaussian uncorrelated time series has exponential tails ($\sim \exp(-4\alpha^2)$), while the daily DAX time series has power-law tails with exponent $\simeq$3.5.

scale that the price y_i will be greater than the y_{i-1} again up to specific level. For narrow band processes, it has been shown that the frequency ν_α^+ can be deduced from the underlying joint probability distribution function (PDF) for x and $dx/dt = \dot{x}$. Rice proved that [2]

$$\nu_\alpha^+ = \int_0^\infty \dot{x} p(x = \alpha, \dot{x}) d\dot{x}, \tag{1}$$

where $p(x, \dot{x})$ is the joint PDF of x and $\dot{x}$. For discrete time series (of course all of real data are discrete), the frequency ν_α^+ can be written in terms of joint cumulative probability distribution, $P(x_i > \alpha, x_{i-1} < \alpha)$ as [39],

$$\nu_\alpha^+ = P(x_i > \alpha, x_{i-1} < \alpha) = \int_{-\infty}^{\alpha} \int_{\alpha}^{\infty} p(x_i, x_{i-1}) dx_i dx_{i-1}, \tag{2}$$

where $p(x_i, x_{i-1})$ is the joint PDF of x_i and x_{i-1}. The inverse of frequency ν_α^+ gives the average time scale T_α that one should wait to observe the given level α again.

The rest of this paper is organized as follows. Section 2 is devoted to summary of level cross analysis of DAX and daily oil price log-returns. The inverse statistics of DAX and Oil price time series are given in Section 3. Section 4 closes with a discussion and conclusion of the present results.

2. Level crossing

Here, at first we provide the results of level crossing analysis for two normalized log-return time series, daily German stock market index (the DAX) and daily oil price. The daily fluctuations in the oil price and DAX time series were belong to the period 1998–2009. We also study the asymmetric properties of level crossing analysis for positive and negative level crossing of the time series. To have a comparison, we provide also level crossing analysis of synthesized uncorrelated noise. Also we will provide the results of level crossing analysis of high frequency data for DAX with sample rate 4 (1/min), where we have used 2511,000 data points and belongs to the period 1994–2003.

Fig. 1 shows the frequency ν_α^+ for daily log-returns of DAX and synthesized uncorrelated noise. The PDF of synthesized uncorrelated noise is Gaussian and it has white noise nature (i.e. its correlation has delta-function behavior). As shown in Fig. 1, their level crossing frequencies are almost similar near to $\alpha \simeq 0$ and have deviation for levels in the tails. The difference is related to the non-Gaussian PDF of DAX log-return time series (see below). For the normalized Gaussian uncorrelated noise, one can show that the frequency ν_α^+ is given by

$$\nu_\alpha^+ = \frac{1}{4}[1 - erf^2(\alpha/\sqrt{2})], \tag{3}$$

where $erf(U)$ is the error function. In Fig. 1, a comparison between the analytical and numerical results for uncorrelated Gaussian time series is given. For Gaussian uncorrelated data, the frequency ν_α^+ behaves as

$$\nu_\alpha^+ \simeq \frac{1}{4} \exp(-\alpha^2/2\pi) \quad \text{for}\ \alpha \to 0$$