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Empirical shape function of limit-order books in the Chinese stock market

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ABSTRACT

We have analyzed the statistical probabilities of limit-order book (LOB) shape through building the book using the ultra-high-frequency data from 23 liquid stocks traded on the Shenzhen Stock Exchange in 2003. We find that the averaged LOB shape has a maximum away from the same best price for both buy and sell sides of the LOB. The LOB shape function has nice exponential form in the right tail. The buy side of the LOB is found to be abnormally thicker for the price levels close to the same best although there are much more sell orders on the book. We also find that the LOB shape functions for both buy and sell sides have periodic peaks with a period of five. The 1-min averaged volumes at fixed tick level follow log-normal distributions except for the left tails which display power-law behaviors, exhibit abnormal intraday patterns with increasing trend, and possess long memory that cannot be explained by the intraday patterns. Academic implications of our empirical results are also briefly discussed.

1. Introduction

In an order-driven market, limit-order book (LOB) is a queue of orders waiting to be executed and it is the base of continuous double auction mechanism. Orders in the book are sorted according to *price-time priority*. The construction of LOB is a dynamic process. Effective limit orders whose prices do not penetrate the opposite best price are stored in the book, while an effective market order with the price penetrating the opposite best immediately causes a transaction and removes the corresponding orders in the opposite book. In addition, cancelations can also remove the orders in the LOB.

Price levels in the limit-order book are discrete. The difference between two adjacent price levels is the tick size u. It is 0.01 RMB for all stocks in the Chinese market. The price level Δ at any given time t can be defined as follows

$$\Delta = \begin{cases} (p_b - p)/u + 1 & \text{for buy orders} \\ (p - p_a)/u + 1 & \text{for sell orders,} \end{cases} \tag{1}$$

where p is an allowed price in the LOB and p_b and p_a are the best bid and best ask, respectively. According to the definition, $\Delta = 1$ stands for the position at the best bid (ask) in the buy (sell) LOB. Denote $V_b(\Delta, t)$ (respectively $V_s(\Delta, t)$) as the volume

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at level Δ in the buy (respectively sell) LOB at event time t. $V_b(\Delta, t)$ and $V_s(\Delta, t)$ can be viewed as the instant LOB shape functions on the buy and sell sides, respectively.

The LOB shape function is of crucial importance in the research of market microstructure theory of order-driven markets. A brief discussion is in order. The shape of the LOB affects a trader's strategy and thus influences order aggressiveness [1]. Also, the LOB shape determines the virtual price impact. The price impact $I(\omega)$ of a virtual market order of size ω can be determined as follows [2–4]

$$I(\omega) = u \times \sup\left\{ n : \sum_{\Delta=1}^{n} V(\Delta, t) \leqslant \omega \right\}. \tag{2}$$

It is found that the virtual price impact is much stronger than the actual impact [4] and large price fluctuations are not necessarily caused by large orders but rather the liquidity [5,6]. It is rational that a large trader prefers to split his large order and submit when the opposite LOB is thick such that the price does not change much. In contrast, an impatient small trader might submit a small order when the opposite LOB is thin for small Δ's, since usually he does not have ensuing orders. The optimal trading strategy of a large order also depends on the average LOB shape [7,8], which could be improved if one considers the instant LOB shape function rather than the average.

When we want to investigate the topics above analytically, the LOB shape function is usually treated as being continuous. In the derivation of an optimal execution strategy, many unrealistic LOB shape functions have been proposed [7,8]. This makes the framework less useful in practice and calls for a realistic shape function. Indeed, the empirical LOB shape function has been investigated in different stock markets. Bouchaud et al. found that the LOB shape of individual liquid stocks on the Paris Bourse (February 2001) is symmetrical for buys and sells and has a maximum away from the current best bid or ask ($\Delta = 1$) [9]. They also found that the distribution of order size at the best bid or ask can be fitted by a gamma distribution [9]. Potters and Bouchaud investigated three stocks traded on the Nasdaq Stock Market and found that all the LOB shape functions are buy/sell symmetric and only one stock reaches a maximum before relaxation [10]. Similar results on the shape function are also reported using other market data [2–4,11].

In this paper, we shall study in detail the LOB shape of 23 liquid stocks traded on the Shenzhen Stock Exchange (SZSE) in China. The rest of the paper is organized as follows. In Section 2, we describe briefly the database we adopt. Section 3 introduces the average shape of buy and sell sides of the LOB. We then discuss in Section 4 the probability distributions and time dependency of volumes at the first three tick levels. The last section concludes.

2. Data sets

The Chinese stock market is a pure order-driven market where orders are matched resulting in transactions. Our data contain ultra-high-frequency data[1] of 23 liquid stocks listed on the Shenzhen Stock Exchange in 2003 [13]. We find that the results for different stocks are qualitatively similar. Hence we will present the results only for a very liquid stock. In 2003, only limit orders were allowed to submit and the market constituted opening call auction, cooling period and continuous double auction. We focus on the LOB in continuous double auction.

As an example, our presentation is based on the order flow data for a stock named Shenzhen Development Bank Co., LTD (code 000001),[2] whose time stamps are accurate to 0.01 s including details of every event, with the information containing date, order size, limit price, time, best bid, best ask, transaction volume, and aggressiveness identifier (which identifies whether a record is a buy order, a sell order, or a cancelation). The database totally records 3,925,832 events, including 1,718,156 buy orders, 1,595,961 sell orders, 598,750 cancelations and 12,965 invalid orders. Using this nice database, we can rebuild the LOB according to the trading rules [14] and study the statistical probabilities of LOB shape.

3. Averaged shape

In the continuous double auction mechanism, order placement adds volume to the book, while order cancelation or transaction removes volume from the book. It is clear that these three types of events (order placement, order cancelation and transaction) can change the shape of the LOB. In what follows we use event time, not clock time. In this way, the event time t advances by 1 when an event occurs. At every time t, we have an instant LOB shape $V_{b,s}(\Delta, t)$ on each side (buy or sell). The averaged shape of the buy (sell) LOB can be calculated as follows:

$$V_{b,s}(\Delta) = \frac{1}{M} \sum_{t=1}^{M} V_{b,s}(\Delta, t), \tag{3}$$

where M is the number of total events in 2003 for the stock we analyzed.

[1] According to Robert F. Engle, "ultra-high-frequency data is defined to be a full record of transactions and their associated characteristics" and is associated with the "limiting frequency" that is achieved when all transactions are recorded [12]. However, the sampling frequency at the transaction level is actually not the limiting frequency. Our data set contains the flows of orders and their associated characteristics, which has higher frequency. We nevertheless term our data as ultra-high-frequency data.

[2] Each Chinese stock has a unique numeric symbol (stock code) that is the standard and official ticker symbol and is used in trading softwares. The full company name corresponding to a code can be retrieved from the web site of the SZSE at http://www.szse.cn.