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An application of fuzzy methods to evaluate a patent under the chance of litigation

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ABSTRACT

The value of a patent, including such unmeasurable things like the chance of litigation, is calculated using a methodology which combines real option theory and fuzzy numbers. The vagueness about the patent holder's future profits, the validity and scope of the patent, the litigation costs, the court's decision under imperfect enforcement of property rights are specified introducing fuzzy numbers. This method is embedded in a real option computation, where the value of a patent includes the option value of litigation. We study how the value of a patent is affected by the timing and incidence of litigation. The main results are compared with the empirical findings of previous results.

1. Introduction

In the recent debate on patent reform the urgency to explicitly recognize the policy dimension of legal uncertainty is a central issue. Virtually, all property rights contain some elements of uncertainty and both litigation and settlement decisions occur in an environment characterized by imprecise information (Bebchuk, 1984; Farrell & Shapiro, 2008; Shavell, 1989). Fuzzy boundaries of the patent property right seem to be a main cause of the recent explosion of patent litigation (Bessen & Meurer, 2005, 2007, 2008). Recently, the European Commission has started studying the feasibility of alternative schemes against patent litigation risks, including insurance, in view of the dramatic explosion of patent litigation (CJA Ltd., 2006). As far as patent policy is concerned, it is emphasized that the effects of uncertainty should be incorporated in regulation and enforcement rules. This issue requires an accurate quantitative determination of the patent value. In this paper we propose a new comprehensive model, focusing on the different sources of uncertainty. Our main result is a valuation formula for patents which can be used for practical applications.

A patent is usually defined as a right to make exclusive use of an innovation at a predetermined cost for a predetermined period of time, i.e. the life of the patent. As such it is viewed as a real option (see Dixit & Pindyck, 1994). The patent holder may commercialize some products or licence her technology or use it for further developments. The interpretation of patents as real options presupposes an enforceable property right. Yet, an increased number of patents have registered a high frequency of disputes and litigation involving patent holders and alleged infringers, so that the risk that a patent will be declared invalid is substantial. There is a wide variation across patents in their exposure to risk: as Lanjouw and Schankerman (2001) and Allison and Lemley (1998) have shown through detailed empirical evidence, for high-value patents and specific types of patentees the litigation risk can be quite high, in some cases almost offsetting what would otherwise be the R&D incentive provided by patent ownership. Bessen and Meurer (2008) in a most comprehensive empirical research have found that technology that rely heavily on software are vexed by huge patent litigation costs, so that "the patent system has turned from a source of net subsidy to R&D to a net tax" (2008, p. 1). Moreover, "roughly half of all litigated patents are found to be invalid, including some of great commercial significance" (Lemley & Shapiro, 2005, p. 76). Thus, because of uncertainty in the enforcement of property rights, it has been stated that "a patent does not confer upon its owner the right to exclude but rather a right *to try* to exclude by asserting the patent in court" (Lemley & Shapiro, 2005, p. 75). Accordingly, the clarification of the norms about intellectual property right has been indicated as the main challenge for lawyers and politicians in the next decades (Landes & Posner, 2003).

Because of imperfect enforcement of property rights, most patents represent highly uncertain or probabilistic property rights. Lemley and Shapiro (2005) use the term *probabilistic patents*. Modeling patents as probabilistic rights requires to rethink how to reform the patent granting process and the patent litigation procedures.

In this paper we translate the vague notion of probabilistic patents into a mathematical model, where the valuation of patents can be performed by a combination of real options and a fuzzy

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methodology. In order to capture the notion of vagueness about the validity and scope of patents under a regime of imperfect enforcement of property rights, we introduce a promising concept of uncertainty, alternative to probability theory, through the theory of fuzzy sets. In this way, we are able to capture the vague and imprecise ideas the patent holder possesses about her future profits, the validity of the patent, the litigation costs and the court's decision. Moreover, we embed such methodology within a real option approach, where the value of a patent includes the option value of litigation.

There are various papers applying the theory of real options to the valuation of patents although very few of them introduce the patent enforcement process explicitly. Pakes (1986) first estimated the distribution of the returns earned from holding patents as options which are renewed at alternative ages and require renewal fees. Bloom and Van Reenen (2002) builds on Pakes (1986) and derives empirical predictions on the relationship between patents and market uncertainty. Schwartz (2004) implements a simulation model to value patents as complex options, taking into account uncertainty in the cost-to-completion of the project and the possibility of abandoning the project. Takalo and Kanniainen (2000) and Lambrecht (2000) investigate the patenting decision under technological and market uncertainty with competing firms. None of the above-mentioned papers introduce the risk of litigation. To the best of our knowledge, the only analyses of the option value of litigation are Marco (2005) and Baecker (2007). However, Marco (2005) is mainly focused on the timing and incidence of patent litigation and is concerned with the empirical estimates of patent litigation. Baecker (2007) develops at length both the theory and the numerical implementation of some jump-diffusion models, where the risk of litigation is exogenously given and negatively affects the value of the patent in the form of discontinuities or jumps in the value process. He also addresses some issues of endogenous patent risk through a model where the patent holder possesses full knowledge about the probability distribution of the litigation risk.

Our paper is the first that combines a real option to litigate with a fuzzy valuation. The need for a fuzzy valuation comes from the common observation that patent claims are sometimes vaguely defined, the outcomes of a trial are difficult to forecast, legal costs are not easily predictable, it may be years before litigation is concluded, there may be divergence in parties' expectations about the court decision and future cash flows from commercialization are imprecise. Although the existing literature has identified the main determinants of litigation, it has not investigated how the value of a patent is affected by the timing and incidence of litigation under an appropriate framework of uncertainty. Section 2 presents the model of a patent under imperfect enforcement of property rights, where the relevant parameters are fuzzy. The model is solved analytically for infinitely lived patents and in Section 3 the main results are compared with the empirical findings of previous studies. Finally, in Section 4 some concluding remarks are presented.

2. The valuation method

Let us introduce the basic fuzzy valuation method that is used in our application. Here we give the essential definitions and refer to the classical textbooks on fuzzy-stochastic theory, starting from Zadeh (1985). A fuzzy number is a fuzzy set (depicted with tilde) of the real line R, which is commonly defined by a normal, upper semicontinuous, fuzzy convex membership function $\mu : R \to [0,1]$ of compact support. Fuzzy numbers can be interpreted as possibility distributions. While in classical set theory an element either belongs to a set or does not belong to a set, in fuzzy set theory we allow for a degree of membership, that can describe vague, imprecise or inaccurate knowledge about some estimates or data.

The γ-cut of a fuzzy number is given by: $\tilde{\mu}_\gamma = \{x \in R | \tilde{\mu}(x) \geqq \gamma\}$, $\gamma \in (0,1]$, and $\tilde{\mu}_0 = cl\{x \in R | \tilde{\mu}(x) \geqq 0\}$, where cl denotes the closure of an interval. Let us write the closed intervals as $\tilde{\mu}_\gamma = \left[\tilde{\mu}_\gamma^-, \tilde{\mu}_\gamma^+\right]$ for $\gamma \in [0,1]$, so that $\tilde{\mu}_\gamma^-$ denotes the left-hand side and $\tilde{\mu}_\gamma^+$ the right-hand side of the γ-cut. Given two fuzzy numbers, $\tilde{\mu}$ and $\tilde{\eta}$, the partial order $\succsim$ on fuzzy numbers can be defined such that $\tilde{\mu} \succsim \tilde{\eta}$ means that $\tilde{\eta}_\gamma \subset \tilde{\mu}_\gamma$ for all $\gamma \in [0,1]$ (see Yoshida, 2002; Ogura & Li, 1996). The arithmetic operations on two fuzzy numbers are defined in the standard way (see, for example, Kaufmann, 1984) in terms of the γ-cuts for $\gamma \in [0,1]$. In particular, for fuzzy numbers $\tilde{\mu}$ and $\tilde{\eta}$ the addition and subtraction $\tilde{\mu} \pm \tilde{\eta}$ and the scalar multiplication $a\mu$, where $a \geqslant 0$, are fuzzy numbers:

$$(\tilde{\mu}+\tilde{\eta})_\gamma = \left[\tilde{\mu}_\gamma^- + \tilde{\eta}_\gamma^-, \tilde{\mu}_\gamma^+ + \tilde{\eta}_\gamma^+\right],$$
$$(\tilde{\mu}-\tilde{\eta})_\gamma = \left[\tilde{\mu}_\gamma^- - \tilde{\eta}_\gamma^+, \tilde{\mu}_\gamma^+ - \tilde{\eta}_\gamma^-\right],$$
$$(a\tilde{\mu})_\gamma = \left[a\tilde{\mu}_\gamma^-, a\tilde{\mu}_\gamma^+\right].$$

Moreover, multiplication between two fuzzy numbers $\tilde{\mu}$ and $\tilde{\eta}$ is obtained as follows:

$$(\tilde{\mu}\tilde{\eta})_\gamma = \left[(\tilde{\mu}\tilde{\eta})_\gamma^-, (\tilde{\mu}\tilde{\eta})_\gamma^+\right],$$

where

$$(\tilde{\mu}\tilde{\eta})_\gamma^- = \min\left[\tilde{\mu}_\gamma^-\tilde{\eta}_\gamma^-, \tilde{\mu}_\gamma^-\tilde{\eta}_\gamma^+, \tilde{\mu}_\gamma^+\tilde{\eta}_\gamma^-, \tilde{\mu}_\gamma^+\tilde{\eta}_\gamma^+\right]$$

and

$$(\tilde{\mu}\tilde{\eta})_\gamma^+ = \max\left[\tilde{\mu}_\gamma^-\tilde{\eta}_\gamma^-, \tilde{\mu}_\gamma^-\tilde{\eta}_\gamma^+, \tilde{\mu}_\gamma^+\tilde{\eta}_\gamma^-, \tilde{\mu}_\gamma^+\tilde{\eta}_\gamma^+\right].$$

Finally, division between two fuzzy numbers $\tilde{\mu}$ and $\tilde{\eta}$ is given by:

$$\left(\frac{\tilde{\mu}}{\tilde{\eta}}\right)_\gamma = \left[\left(\frac{\tilde{\mu}}{\tilde{\eta}}\right)_\gamma^-, \left(\frac{\tilde{\mu}}{\tilde{\eta}}\right)_\gamma^+\right],$$

where

$$\left(\frac{\tilde{\mu}}{\tilde{\eta}}\right)_\gamma^- = \min\left[\frac{\tilde{\mu}_\gamma^-}{\tilde{\eta}_\gamma^-}, \frac{\tilde{\mu}_\gamma^-}{\tilde{\eta}_\gamma^+}, \frac{\tilde{\mu}_\gamma^+}{\tilde{\eta}_\gamma^-}, \frac{\tilde{\mu}_\gamma^+}{\tilde{\mu}_\gamma^+}\right] \quad \text{and} \quad \left(\frac{\tilde{\mu}}{\tilde{\eta}}\right)_\gamma^+ = \max\left[\frac{\tilde{\mu}_\gamma^-}{\tilde{\eta}_\gamma^-}, \frac{\tilde{\mu}_\gamma^-}{\tilde{\eta}_\gamma^+}, \frac{\tilde{\mu}_\gamma^+}{\tilde{\eta}_\gamma^-}, \frac{\tilde{\mu}_\gamma^+}{\tilde{\mu}_\gamma^+}\right].$$

Let (Ω, Π, A) be a probability space. A fuzzy-number-valued map $\widetilde{X}$ is called a fuzzy random variable (see, for example, Ogura & Li, 1996) if $\{(\omega, x) \in \Omega \times R | \widetilde{X}(\omega)(x) \geqslant \gamma\}$ is measurable for all $\gamma \in [0,1]$. It is called integrably bounded if both $\omega \to \widetilde{X}^-(\omega)$ and $\omega \to \widetilde{X}^+(\omega)$ are integrable for all $\gamma \in [0,1]$. The expectation $E(\widetilde{X})$ of the integrably bounded fuzzy random variable is also defined by a fuzzy number

$$E(\widetilde{X})(x) = \sup_{\gamma \in (0,1)} \min\left\{\gamma, 1_{H(\widetilde{X})_\gamma}(x)\right\}, \quad x \in R,$$

where

$$H(\widetilde{X})_\gamma = \left[\int_\Omega \widetilde{X}_\gamma^-(\omega) d\Pi(\omega), \int_\Omega \widetilde{X}_\gamma^+(\omega) d\Pi(\omega)\right], \quad \gamma \in [0,1].$$

Let us introduce the valuation method (along the line in Agliardi & Agliardi, 2009, 2011), which is based on fuzzy variables. Suppose an innovator owns a patent allowing her to generate additional cash flows from commercializing some product. For simplicity, let us assume that the protection period is infinite, which facilitates the derivation of a closed-form solution. Commercialization is related with some expected income which fluctuates randomly. This income can be either in the form of royalties or in the form of increased revenues from the ability to exclude others from the market. Let P denote the net cash flow resulting from the patent, which is described by the following stochastic dynamics: