Physica A 391 (2012) 1469–1480

Contents lists available at SciVerse ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Pricing European option with transaction costs under the fractional long memory stochastic volatility model☆

Xiao-Tian Wang [a,*], Min Wu [a], Ze-Min Zhou [b], Wei-Shu Jing [a]

[a] Department of Mathematics, South China University of Technology, Guangzhou, 510640, Guangdong, PR China

[b] Information College, China Renmin University, 100872, PR China

ARTICLE INFO

Article history:
Received 11 January 2010
Received in revised form 19 July 2011
Available online 20 November 2011

Keywords:
Anchoring-adjustment
Reference point effect
Delta-hedging
Scaling
Transaction costs

ABSTRACT

This paper deals with the problem of discrete time option pricing using the fractional long memory stochastic volatility model with transaction costs. Through the 'anchoring and adjustment' argument in a discrete time setting, a European call option pricing formula is obtained.

1. Introduction

Over the last few years, the financial markets are regarded as complex and nonlinear dynamic systems. A series of studies have found that many financial market time series display scaling laws and that there exist the excess kurtosis and skewness in the stock price returns. It is also well known that the returns and the volatilities of the stock prices often exhibit the long memory property where the autocorrelation of the returns and the absolute and squared returns of time series are characterized by a very slow decay. These features are the crucial components of asset risk management, investment portfolios and option pricing, for their presences are closely connected to the predictability of the return and volatility. Therefore, it has been proposed that one should replace the Brownian motions in the classical Black–Scholes model [1] and the Hull–White stochastic volatility model [2] by two processes with long-range dependence. A simple modification, named the fractional long memory stochastic volatility model, is to introduce fractional Brownian motion (fBm) as the source of randomness. Thus, one adds two parameters, H and H_1, in the Hull–White-like stochastic volatility model, to model the dependence structures in the stock returns and the volatilities. The fractional long memory stochastic volatility model can capture the excess kurtosis and skewness of the stock price returns and the long-range dependence in the stock returns and volatilities.

In this paper, on the basis of the points of view of behavioral finance [3–6] and econophysics [7–11] and quantum mechanics [12] and empirical findings of the long-range dependence in stock returns and volatilities by Zenyl Abidin Ozdemir [13], Ramirez et al. [14], Tabak et al. [15], Erzgraber [16], Mariani [17], Daniel et al. [18], Duan et al. [19], Daniel et al. [20], Willinger et al. [21], Thomas Lux and Micheal Marchesi [22], Ding et al. [23], Lobato and Savin [24], Bay and Tsay [25], Floresu and Marimani [26] we will study the option pricing problem under transaction costs while the dynamics of stock price S_t satisfies.

☆ This work is supported by the National Natural Science Foundation of China (11071082-10771075).

* Corresponding author.
E-mail address: swa001@126.com (X.-T. Wang).

0378-4371/$ – see front matter © 2011 Elsevier B.V. All rights reserved.
doi:10.1016/j.physa.2011.11.014

$S_t = S_0 \exp(\mu t + \int_0^t \sigma_\tau \mathrm{d}B_H(\tau))$, where $\mu, H \in [\frac{1}{2}, 1)$, and $S_0 > 0$ are constants; $\sigma_t = \sigma_0 \exp(\alpha t + \beta W_{H_1}(t))$; $\sigma_0 > 0$, $\beta > 0$ and α are constants; Hurst exponent $H_1 > H \geq \frac{1}{2}$; the fBm $B_H(t)$ is independent of the fBm $W_{H_1}(t)$ and $\int_0^t \sigma_\tau \mathrm{d}B_H(\tau)$ is a pathwise integral with respect to the $B_H(\tau)$.

Leland [27] was the first who examined option replication in the presence of transaction costs in a discrete time setting. From the point of view of Leland [27], in a model where transaction costs are incurred at every time the stock or the bond is traded, the arbitrage-free argument used by Black–Scholes [1] no longer applies. The problem is that due to the infinite variation of the geometric Brownian motion, perfect replication incurs an infinite amount of transaction costs. Hence, he suggested a delta hedging strategy incorporating transaction costs based on revision at a discrete number of times.

In Section 2 by using a delta-hedging strategy, initiated by Leland [27], we will show that the price of European options with transaction costs under the fractional long memory stochastic volatility model is determined by the trading time intervals. In Section 3, the reference point effect is discussed. In Section 4, a conclusion is given.

2. A European option pricing model for a fractional economy under transaction costs

Since traders are bounded rational, they either underreact or continuing overreact to news. This behavior may lead to the features of "leptokuric" and long-range dependence in stock return time series. The fractional Brownian motion may be a useful tool for capturing this phenomenon.

Let $\left(\Omega, F, (\mathcal{F}_t)_{0\leq t\leq T}, P\right)$ be a complete probability space on which two standard fractional Brownian motions $B_H(t)$, and $W_{H_1}(t)$ with Hurst exponent H and H_1 are defined, i.e. $B_H(t)$, and $W_{H_1}(t)$, are continuous, centered Gaussian processes with covariance functions [8]

$$\operatorname{cov}(B_H(t), B_H(s)) = \frac{1}{2}\left(|t|^{2H} + |s|^{2H} - |t-s|^{2H}\right), \quad s, t \in R;$$

and

$$\operatorname{cov}\left(W_{H_1}(t), W_{H_1}(s)\right) = \frac{1}{2}\left(|t|^{2H_1} + |s|^{2H_1} - |t-s|^{2H_1}\right), \quad \text{s,t} \in R.$$

In particular, we assume that

$$B_H(t) = \frac{1}{\Gamma\left(H+\frac{1}{2}\right)}\left[\int_{-\infty}^{0}\left[|t-\tau|^{H-\frac{1}{2}} - |\tau|^{H-\frac{1}{2}}\right]\mathrm{d}B(\tau) + \int_0^t |t-\tau|^{H-\frac{1}{2}}\mathrm{d}B(\tau)\right], \quad t \in R,$$

where $B(t)$ is a standard Brownian motion process, and for $t < 0$, the notation $\int_0^t$ should be interpreted as $-\int_t^0$.

By $\{\mathcal{F}_t\}_{t\in[0,T]}$ we denote the filtration generated by $B_H(t)$ and $W_{H_1}(t)$, augmented with P-null sets and made right-continuous. Here we assume that $\mathcal{F} = \mathcal{F}_T$ for some $T \in (0, +\infty)$, $\mathcal{F}_0$ is trivial, and P is the real world probability.

It can be easily seen that $E\left(B_H(t) - B_H(s)\right)^2 = |t-s|^{2H}$. Furthermore, $B_H(t)$ has stationary increments and is H-self-similar; that is, for all $a > 0$, $B_H(at)_{t\in R}$ has the same distribution as $\left(a^H B_H(t)\right)_{t\in R}$; and for all $\delta t > 0$, $(B_H(t+\delta t) - B_H(t))_{t\in R}$ has the same distribution as $\left[a^{-H}\left(B_H(t+a\delta t) - B_H(t)\right)\right]_{t\in R}$.

Let

$$\xi(n) = B_H(n+1) - B_H(n), n = 0, 1, 2, \ldots,$$
$$r(n) = E[\xi(0)\xi(n)], \quad n = 0, 1, 2, \ldots.$$

Then

$$r(n) = \frac{1}{2}[(n+1)^{2H} - 2n^{2H} + (n-1)^{2H}],$$
$$r(n) \sim H(2H-1)n^{2H-2} \quad \text{as } n \to \infty, \text{ if } H \neq \frac{1}{2}.$$

And

$$r(n) = 0, \quad n \geq 1, ifH = \frac{1}{2}.$$

Hence (a) if $0 < H < \frac{1}{2}$, then $\sum_{i=0}^{\infty} |r(n)| < \infty$;

(b) if $H = \frac{1}{2}$, $\{\xi(n)\}$ is independent; and the corresponding fractional Brownian motion is the usual standard Brownian motion.

(c) if $\frac{1}{2} < H < 1$, then $\sum_{n=0}^{\infty} |r(n)| = \infty$.

Actually, if $0 < H < \frac{1}{2}$, $r(n) < 0$ for $n \geq 1$ ($\{\xi(n)\}$ is negative correlation);

and if $\frac{1}{2} < H < 1$,$r(n) > 0$ for $n \geq 1$ ($\{\xi(n)\}$ is positive correlation).

The property $\sum |r(n)| = \infty$ is often referred to as long-range dependence and is especially of interest in finance (e.g., see [21] and references therein).

In Ref. [21], Willinger et al. have found empirical evidence of long-range dependence in stock price returns and points out that long-range dependence is widespread in economics and finance and has remained a topic of active research. On