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Credit derivatives with multiple debt issues ☆

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Abstract

We evaluate the most actively traded types of credit derivatives within a unified pricing framework that allows for multiple debt issues. Since firms default on all of their obligations, total debt is instrumental in the likelihood of default and therefore in credit derivatives valuation. We use a single factor interest rate model where the exponential default frontier is based on total debt and is made coherent with observed bond prices. Analytical formulae are derived for credit default swaps, total return swaps (both fixed-for-fixed and fixed-for-floating), and credit risk options (CROs). Price behaviors and hedging properties of all these credit derivatives are investigated. Simulations document that credit derivatives prices may be significantly affected by terms of debt other than those of the reference obligation. The analysis of CROs indicates their superior ability to fine-tune the hedging of magnitude and arrival risks of default.

JEL classification: G12; G13
Keywords: Credit derivatives; Structural models; Debt structure; Options; Swaps

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Credit derivatives are financial contracts that allow one party to transfer credit risk to another party. They permit the trading of credit risk separately from other sources of risk. The market for credit derivatives, with New York and London being its most active places, is estimated between $400 billion and $1 trillion notional outstanding according to Hargreaves (2000). Its exponential growth within the last ten years has retained the attention of many researchers in credit risk pricing. Contributions are commonly classified the same way as the literature on corporate bond pricing.

"Reduced form" models (or "intensity" models) directly specify the dynamics of the bond price and view default as an unpredictable event (e.g. the first jump of a Poisson process). This approach, initiated by Jarrow and Turnbull (1995), has been extended to credit derivatives by Flesaker et al. (1994), Kijima and Komoribayashi (1998), Duffie and Singleton (1999), or Chen and Sopranzetti (2002).

By contrast, "structural" models (or "firm value" models) specify the dynamics of the firm assets value and use contingent claims analysis to price the securities issued by this firm. This is the approach we follow here. The seminal reference is Merton (1974) yielding a solution to the price of a corporate discount bond. Extensions of this work evaluate more sophisticated debt contracts (see e.g. Black and Cox (1976) for debt with a safety covenant; Geske (1977) for coupon-bearing debt, or Brennan and Schwartz (1980) for convertible debt). Other extensions evaluate debt contracts under more complex default rules (see e.g. Leland (1994) where the decision to default is driven by the capital structure static trade-off; Anderson and Sundaresan (1996) where debt is subject to strategic default, or François and Morellec (2002) where default leads to a Court-supervised reorganization procedure). Applications of structural models to credit derivatives include the works of Longstaff and Schwartz (1995a), Das (1995), Pierides (1997) and Ammann (2001). All these papers mostly study credit risk options (referred to as CROs). The model of Longstaff and Schwartz (1995a) relies on the empirically supported assumption that credit spreads are mean-reverting. Their model values European CROs that pay the credit spread of a bond at a given date. However, this pay-off is not triggered by the event of default, and the option is therefore inadequate for hedging purposes. Das (1995) derives a closed-form formula in continuous time for the value of a CRO written on a bond paying a continuous coupon. Interest rate is assumed constant. He further develops a discrete time model with a Heath–Jarrow–Morton term structure model, but no analytical result is available. Das and Sundaram (2000) work in that same framework, but directly model the credit spread. Hence they value the same kind of CROs as in Longstaff and Schwartz (1995a). Ammann (2001) adopts a compound option approach to value CROs written on discount bonds. In these three papers, default is exogenous. By contrast, Pierides (1997) values CROs with an endogenous default rule at the cost of assuming that interest rates are constant. Recently, Bélanger et al. (2001) have developed a general pricing framework for credit derivatives that embed both structural and reduced form models.

With respect to the literature on contingent claims models for credit derivatives, our contribution consists of four points. First, we provide closed form formulae not only for CROs but also for all other most actively traded credit derivatives within a