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Equilibrium dynamics in the one-sector endogenous growth model with physical and human capital

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Abstract

This paper concerns the transitional dynamics of the one sector endogenous growth model with physical and human capital when gross investments are irreversible. It has been claimed that the transition path is on the stable saddle path of the system that describes the dynamics of the economy as long as the constraint on nonnegative gross investment in one of the factors is binding. We show that this does not have to be the case. The dynamics can be determined by noting that the continuity of the shadow prices involves the continuity of the consumption path.

JEL classification: O40

Keywords: Endogenous growth; Transitional dynamics

1. Introduction

The one sector model with physical and human capital is among the simplest models of endogenous growth. If investments in physical and human capital are reversible, the model features no transitional dynamics: The ratio of physical to human capital jumps immediately to the optimal value, in which the net returns on each type of capital are equalized, and the economy is on a balanced growth path after that. Transitional dynamics arises, however, if investments are irreversible, i.e. must each be nonnegative. Now, there can be an initial phase when one of the nonnegativity constraints on investment is binding for one type of capital.

Barro and Sala-i-Martín (1995) claim that during the transition the economy will be on the stable saddle path corresponding to the system that describes the dynamics of

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the economy as long as the constraint on nonnegative investment in one of the factors is binding. We shall show that this does not have to be case. Instead, transitional dynamics can be determined by noting that the continuity of the shadow prices involves the continuity of the consumption path. Thus, the transition path has to be chosen such that when the balanced growth path is reached, consumption reaches its balanced growth value without jumping. This transition path does not have to be on the stable saddle path of the initial dynamical system.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 performs a phase diagram analysis. Section 4 shows that the transition path does not have to be on the stable saddle path of the initial dynamical system. Section 5 explains how the true transition path can be computed. Section 6 concludes.

2. The model

Consider a closed economy inhabited by an infinitely lived representative household that maximizes the intertemporal utility function $\int_0^\infty e^{-\rho t}(c^{1-\theta}-1)/(1-\theta)\,dt$, where c denotes consumption, $\rho > 0$ is the rate of time preference and $1/\theta > 0$ is the elasticity of intertemporal substitution. Output y is produced with the Cobb–Douglas production function $y = Ak^\alpha h^{1-\alpha}$, $0 < \alpha < 1$, where k is physical capital, and h is human capital. The economy's resource constraint is $y = i_k + i_h + c$, where i_k and i_h are gross investment in physical and human capital, respectively, which must each be nonnegative. The stocks of physical and human capital evolve according to the dynamic equations $\dot{k} = i_k - \delta k$ and $\dot{h} = i_h - \delta h$, respectively.

This model has been analyzed by Barro and Sala-i-Martín (1995, Chap. 5). The current value Lagrangian of the representative household's maximization problem is

$$L = (c^{1-\theta}-1)/(1-\theta) + \lambda(i_k - \delta k) + \mu(i_h - \delta h) + \psi(Ak^\alpha h^{1-\alpha} - i_k - i_h - c)$$
$$+\eta i_k + \xi i_h,$$

where λ and μ are the shadow prices of physical and human capital, ψ is the multiplier associated with the economy's resource constraint, and η and ξ are the multipliers associated with the nonnegativity constraints on gross investment. The necessary conditions are

$$\partial L/\partial c = c^{-\theta} - \psi = 0, \tag{1a}$$

$$\partial L/\partial i_k = \lambda - \psi + \eta = 0, \quad i_k \geqslant 0, \quad \eta \geqslant 0, \quad \eta i_k = 0, \tag{1b}$$

$$\partial L/\partial i_h = \mu - \psi + \xi = 0, \quad i_h \geqslant 0, \quad \xi \geqslant 0, \quad \xi i_h = 0, \tag{1c}$$

$$Ak^\alpha h^{1-\alpha} = i_k + i_h + c, \tag{1d}$$

$$\dot{\lambda} = (\rho+\delta)\lambda - \psi\alpha A k^{\alpha-1}h^{1-\alpha}, \tag{1e}$$

$$\dot{\mu} = (\rho+\delta)\mu - \psi(1-\alpha)Ak^\alpha h^{-\alpha}, \tag{1f}$$

$$\lim_{t\to\infty} e^{-\rho t}\lambda k = \lim_{t\to\infty} e^{-\rho t}\mu h = 0. \tag{1g}$$