Commodity money equilibrium in a convex trading post economy with transaction costs

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Abstract

Existence and efficiency of general equilibrium with commodity money is investigated in an economy where N commodities are traded at $N(N-1)/2$ commodity-pairwise trading posts. Trade is a resource-using activity recovering transaction costs through the spread between bid (wholesale) and ask (retail) prices. Budget constraints, enforced at each trading post separately, imply demand for a carrier of value between trading posts. Existence of general equilibrium is established under conventional convexity and continuity conditions while structuring the price space to account for distinct bid and ask price ratios. Commodity money flows are identified as the difference between gross and net inter-post trades.
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"[An] important and difficult question . . . [is] not answered by the approach taken here: the integration of money in the theory of value . . ."
Gerard Debreu, *Theory of Value* (1959)

1. Introduction

It is well-known that the Arrow–Debreu model of Walrasian general equilibrium cannot account for money. Professor Hahn (1982) writes

"The most serious challenge that the existence of money poses to the theorist is this: the best developed model of the economy cannot find room for it. The best developed model is, of course, the Arrow–Debreu version of a Walrasian general equilibrium. A first, and . . . difficult . . . task is to find an alternative construction without . . . sacrificing the clarity and logical coherence . . . of Arrow–Debreu."

This paper pursues development of foundations for a theory of money based on elaborating the detail structure of an Arrow–Debreu model. The elementary first step is to create a general equilibrium where there is a well-defined demand

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for a medium of exchange—a carrier of value between transactions. This is arranged by replacing the single budget constraint of the Arrow–Debreu model with the requirement that the typical household or firm pays for its purchases directly at each of many separate transactions. Transactions take place at commodity-pairwise trading posts. Then a well-defined demand for media of exchange (commodity monies, not necessarily unique) arises endogenously as an outcome of the market equilibrium. The use of media of exchange is particularly evident when the structure of demands is characterized by an absence of double coincidence of wants (Jevons, 1875). Media of exchange are characterized as the carrier of value between transactions (not fulfilling final demands or input requirements themselves), the difference between gross and net trades[1]. Related general equilibrium models with transaction cost include Foley (1970); Hahn (1971, 1973); Kurz (1974); Starrett (1973); Starr (2003c).

The trading post model is intended to provide a parsimonious[2] addition to the Arrow–Debreu model sufficient to generate a theory of money. The monetary structure of trade is shown to be a consequence of the price theory general equilibrium, not a separate assumption.

1.1. Structure of the trading post model

In the trading post model, transactions take place at commodity-pairwise trading posts (Shapley and Shubik, 1977; Walras, 1874; Wicksell, 1936) with budget constraints (you pay for what you get in commodity terms) enforced at each post. Prices — bid (wholesale) and ask (retail) — are quoted as commodity rates of exchange. Trade is arranged by firms, typically buying at bid prices and selling at ask prices, incurring costs (resources used up in the transaction process) and recouping them through the bid/ask spread. Market equilibrium occurs when bid and ask prices at each trading post have adjusted so that all trading posts clear.

1.2. Structure of the proof

The structure of the proof of existence of general equilibrium follows the approach of Arrow and Debreu (1954); Debreu (1959); Starr (1997). The usual assumptions of continuity, convexity (traditional but by no means innocuous in this context), and no free lunch/irreversibility are used. The price space at a trading post for exchange of one good at bid price for another at ask price is the unit 1-simplex, allowing any possible nonnegative relative price ratio. The price space for the economy as a whole then is a Cartesian product of unit 1-simplices. The attainable set of trading post transactions is compact. As in Arrow and Debreu (1954), the model considers transaction plans of firms and households artificially bounded in a compact set including the attainable set as a proper subset. Price adjustment to a fixed point with market clearing leads to equilibrium of the artificially bounded economy. But the artificial bounds are not a binding constraint in equilibrium. The equilibrium of the artificially bounded economy is as well an equilibrium of the original economy.

1.3. Conclusion: The medium(a) of exchange

The general equilibrium specifies each household and firm's trading plan. At the conclusion of trade, each has achieved a net trade. Gross trades include trading activity that goes to paying for acquisitions and accepting payment for sales rather than directly implementing desired net trades. It is easy to calculate gross trades and net trades at equilibrium. For households, the difference — gross trades minus net trades — represents trading activity in carriers of value between trades, media of exchange (perhaps including some arbitrage). Since firms perform a market-making function within trading posts, identification of media of exchange used by firms is not so straightforward. After netting out intra-post trades, the remaining difference between inter-post gross and net trades represents the firms' trade flows of media of exchange. In some examples (see Starr, 2003a,b, 2008) the medium of exchange may be a single specialized

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[1] The present model is an alternative to the fiat money models of overlapping generations (Wallace, 1980) and of search (Kiyotaki and Wright, 1989). There a unique unbacked fiat money of positive value is typically assumed and presented as a bubble. The models allow, as well, a non-monetary no-trade equilibrium where the fiat money has a value of zero. In the present model, the existence of media of exchange and their values are endogenously determined. It is possible to accommodate in an Arrow–Debreu setting an intrinsically worthless paper money trading at a positive value and used as a common medium of exchange. The rationale is that taxes payable in paper money provide backing for a positive value, and low transaction cost ensures use as medium of exchange (Goldberg, 2005; Smith, 1776; Starr, 2003a,b).

[2] Consistent with Ockham's razor.