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Optimal financing and dividend control of the insurance company with fixed and proportional transaction costs

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ABSTRACT

We consider the optimal financing and dividend control problem of the insurance company with fixed and proportional transaction costs. The management of the company controls the reinsurance rate, dividends payout as well as the equity issuance process to maximize the expected present value of the dividends payout minus the equity issuance until the time of bankruptcy. This is the first time that the financing process in an insurance model with two kinds of transaction costs, which come from real financial market has been considered. We solve the mixed classical-impulse control problem by constructing two categories of suboptimal models, one is the classical model without equity issuance, the other never goes bankrupt by equity issuance.

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1. Introduction

In this paper, we consider an insurance company with the fixed and the proportional transaction costs. In this model the management controls the dividends payout, equity issuance and the risk exposure by proportional reinsurance policy. We study the value of the company via the expected present value of the dividends payout minus the equity issuance. This is a mixed classical-impulse control on diffusion models. Diffusion models for companies that control their risk exposure by means of dividends payout have attracted a lot of interests recently. We refer the readers to He and Liang (2007) and the references therein. Optimizing dividends payout is a classical problem starting from the early work of Borch (1969, 1967) and Gerber (1972). For some applications of control theory in insurance mathematics, see, Harrison and Taksar (1983), Højgaard and Taksar (1998a,b), Martin-Löf (1983), Asmussen and Taksar (1997) and Cadenillas et al. (2006). A survey can be found in Taksar (2000).

However, there are very few results concerned with the equity issuance of the insurance company. In the real financial market, equity issuance is an important approach for the insurance company to earn profit and reduce risk. Harrison and Taksar (1983) consider the optimal control problem with a lower and an upper reflecting barrier. Sethi and Taksar (2002) recently considered the model for the company that can control its risk exposure by issuing new equity as well as paying dividends. He and Liang (2007) work out the optimal financing and dividend control problem of the insurance company without the fixed transaction costs.

In this paper, we consider both the fixed and the proportional transaction costs incurred by the equity issuance. The amount of money paid by the shareholder is $K + \beta_2\xi$, $\beta_2 > 1$, to meet the equity issuance of ξ. K is the fixed transaction costs generated by the advisory and consulting fees, β_2 is the proportional transaction costs generated by the tax. We assume that if the company pays l as dividends, the shareholder can get $\beta_1 l$, $\beta_1 < 1$, and we can omit the fixed transaction costs in the dividends payout process because the financial system is operated with an ever increasing efficiency and the dividends payout processes seldom generate fixed transaction

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costs. We refer the reader to Cadenillas et al. (2006), which consider the optimal dividends policy of the insurance company with the fixed and proportional transaction costs, and without the equity issuance.

Motivated by the work of He and Liang (2007), Harrison and Taksar (1983) and Sethi and Taksar (2002), we can consider the equity issuance and dividends payout as the absorbing and reflecting boundaries of the reserve process, respectively. We will deal with the mixed classical-impulse control problem by using the line of He and Liang (2007). We expect our results would be of interest for theory of mixed classical-impulse control.

The paper is organized as follows: In Section 2, we establish the control model of the insurance company with fixed and proportional transaction costs. In Section 3, we present some mathematical results proved by He and Liang (2007) for proving the main results of this paper. In Section 4, we construct solutions of two categories of suboptimal models. One is the classical model without equity issuance, the other never goes bankrupt by equity issuance. uct solutions of two categories of suboptimal models. In Section 5, we identify the value function and the optimal strategy with the corresponding solution in either category of suboptimal models, depending on the relationships between the coefficients. We give the conclusion of this paper in Section 6.

2. Control model of the insurance company with fixed and proportional transaction costs

To give a mathematical foundation of our model, we fix a filtered probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, P)$. $\{\mathcal{W}_t, t \geq 0\}$ is a standard Brownian Motion on this probability space, $\mathcal{F}_t$ represents the information available at time t, any decision made up to time t is based on $\mathcal{F}_t$. In our model, we denote L_t as the cumulative amount of dividends paid from time 0 to time t. We assume that the process $\{L_t, t \geq 0\}$ is $\{\mathcal{F}_t, t \geq 0\}$-adapted, increasing, right-continuous with left limits and $L^{\pi}_{0-} = 0$. The equity issuance process $\{G^{\pi}\}$ is described by a sequence of increasing stopping times $\{\tau_i, i = 1, 2 \ldots\}$ and a sequence of random variables $\{\xi_i, i = 1, 2 \ldots\}$, which are associated with the time and the amount of the equity issuance. A control policy π is described by stochastic processes $\pi \equiv \{a_\pi; L^\pi; G^\pi\} \equiv \{a_\pi; L^\pi; \tau^\pi_1, \tau^\pi_2 \ldots \tau^\pi_n \ldots; \xi^\pi_1, \xi^\pi_2, \ldots, \xi^\pi_n, \ldots\}$.

For a control policy π, we assume that the liquid reserve of the insurance company evolves according to the stochastic equation,

$$R^\pi_t = x + \int_0^t \mu a_\pi(s)\mathrm{d}s + \int_0^t \sigma a_\pi(s)\mathrm{d}\mathcal{W}_s - L^\pi_t + \sum_{n=1}^{\infty} I_{\{\tau^\pi_n \leq t\}}\xi^\pi_n, \tag{2.1}$$

where $1 - a_\pi(t) \in [0, 1]$ is the proportional reinsurance rate. The company is considered as bankrupt as soon as the reserves fall below 0. We define the time of bankruptcy as $\tau = \inf\{t \geq 0 : R^\pi_t < 0\}$. τ is an $\mathcal{F}_t$-stopping time. If the company issues some equity, the time of bankruptcy could be infinite.

Our main objective is to maximize the expected present value of the dividends payout minus the equity issuance before bankruptcy,

$$J(x,\pi) = \mathbf{E}\left[\int_0^\tau \mathrm{e}^{-cs}\beta_1 \mathrm{d}L^\pi_s - \sum_{n=1}^{\infty} \mathrm{e}^{-c\tau^\pi_n}(K + \beta_2\xi^\pi_n)I_{\{\tau^\pi_n \leq \tau\}}\right], \tag{2.2}$$

$$V(x) = \sup_{\pi\in\Pi} J(x,\pi), \tag{2.3}$$

where $\Pi = \{\pi\}$ denotes the set of all admissible control policies, c denotes the discount rate. In the equity issuance process, $\beta_2 > 1$ is the proportional transaction costs generated by the tax. K is the fixed transaction costs generated by the advisory and consulting fees. In the dividends payout process, $\beta_1 < 1$ is the proportional transaction costs generated by the tax. We solve the optimal mixed classical-impulse control problem (2.2) and (2.3) by finding some conditions on $(\mu, \sigma, c, \beta_1, \beta_2, K)$ such that $V(x)$ satisfies a type of HJB equations. Then we get $V(x)$ and the optimal strategy π^* associated with $V(x)$.

3. Preliminary of the problem

In this section we present two lemmas before proving the main results of this paper in Sections 4 and 5. Since the proofs of the lemmas are completely similar to that of He and Liang (2007), we omit them here.

Lemma 3.1. *Let* $x_0 = \frac{(1-\gamma)\sigma^2}{\mu}$ *and* $m = \frac{\gamma K}{(1-\gamma)\beta_2} < x_0$. *Then there exists a unique* $x_{1*} = x_{1*}(c, \mu, \sigma, \beta_1) > x_0$ *satisfies the following equation in* x_1,

$$\left(\frac{1}{d_-} - \frac{x_0}{\gamma}\right)\frac{\beta_1 d_+}{d_+ - d_-}\mathrm{e}^{d_-(x_0-x_1)} - \left(\frac{1}{d_+} - \frac{x_0}{\gamma}\right)\frac{\beta_1 d_-}{d_+ - d_-}\mathrm{e}^{d_+(x_0-x_1)} = 0, \tag{3.1}$$

where $d_- = \frac{-\mu-\sqrt{\mu^2+2c\sigma^2}}{\sigma^2}$, $d_+ = \frac{-\mu+\sqrt{\mu^2+2c\sigma^2}}{\sigma^2}$, $\gamma = \frac{c}{c+\frac{\mu^2}{2\sigma^2}}$.

Lemma 3.2. *Let* $x_0 = \frac{(1-\gamma)\sigma^2}{\mu}$ *and* $m = \frac{\gamma K}{(1-\gamma)\beta_2} < x_0$. *Then there exists a unique* $x_{1**} = x_{1**}(c, \mu, \gamma, \sigma, \beta_1, \beta_2, K) > x_0$ *satisfies the following equation in* x_1,

$$\frac{1}{\gamma x_0^{\gamma-1}}\left(\frac{\beta_1 d_+}{d_+ - d_-}\mathrm{e}^{d_-(x_0-x_1)} - \frac{\beta_1 d_-}{d_+ - d_-}\mathrm{e}^{d_+(x_0-x_1)}\right) = \frac{\beta_2}{\gamma m^{\gamma-1}}, \tag{3.2}$$

where d_-, d_+ *and* γ *are the same as in Lemma 3.1.*

4. Two categories of suboptimal solutions

In this section, we consider two categories of suboptimal control problems. $\pi_p = \{a_p, L_p, 0\} \in \Pi$ stands for the control process for the company in which equity issuance is not permitted. The associated optimal return function is defined by

$$V_p(x) = \sup_{\pi_p\in\Pi} J(x,\pi_p), \quad \text{for } x \geq 0.$$

Let $\pi_q = \{a_q, L_q, G_q\} \in \Pi$ be the control process for the company with equity issuance procedures, where $G_q = (\tau^{\pi_q}_1, \tau^{\pi_q}_2 \ldots \tau^{\pi_q}_n \ldots; \xi^{\pi_q}_1, \xi^{\pi_q}_2, \ldots, \xi^{\pi_q}_n, \ldots)$. The associated optimal return function is

$$V_q(x) = \sup_{\pi_q\in\Pi} J(x,\pi_q), \quad \text{for } x \geq 0.$$

The company will never go bankrupt in this case.

Since $\pi_p, \pi_q \in \Pi$, it is easy to see that $V(x) \geq \max\{V_p(x), V_q(x)\}$ via (2.3). We will show that the two suboptimal solutions play an important role in constructing the overall optimal control solutions. In next two subsections, we will establish solutions for each category of the control models.

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