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The Copula-GARCH model of conditional dependencies: An international stock market application

Eric Jondeau, Michael Rockinger*

Swiss Finance Institute and University of Lausanne, Lausanne, Switzerland

Abstract

Modeling the dependency between stock market returns is a difficult task when returns follow a complicated dynamics. When returns are non-normal, it is often simply impossible to specify the multivariate distribution relating two or more return series. In this context, we propose a new methodology based on copula functions, which consists in estimating first the univariate distributions and then the joining distribution. In such a context, the dependency parameter can easily be rendered conditional and time varying. We apply this methodology to the daily returns of four major stock markets. Our results suggest that conditional dependency depends on past realizations for European market pairs only. For these markets, dependency is found to be more widely affected when returns move in the same direction than when they move in opposite directions. Modeling the dynamics of the dependency parameter also suggests that dependency is higher and more persistent between European stock markets.

JEL classification: C51; F37; G11

Keywords: Stock indices; International correlation; Dependency; GARCH model; Skewed Student-*t* distribution; Copula function

* Corresponding author. University of Lausanne, Ecole des HEC, Department of Finance and Insurance, 1015 Lausanne, Switzerland. Tel.: +41 2 16 92 33 48; fax: +41 2 16 92 34 35.
E-mail addresses: eric.jondeau@unil.ch (E. Jondeau), michael.rockinger@unil.ch (M. Rockinger).

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1. Introduction

An abundant literature has investigated how the correlation between stock market returns varies when markets become agitated. In a multivariate GARCH framework, for instance, Hamao et al. (1990), Susmel and Engle (1994), and Bekaert and Harvey (1995) have measured the interdependence of returns and volatilities across stock markets. More specifically, Longin and Solnik (1995) have tested the hypothesis of a constant conditional correlation between a large number of stock markets. They found that correlation generally increases in periods of high-volatility of the U.S. market. In addition, in a similar context, tests of a constant correlation have been proposed by Bera and Kim (2002) and Tse (2000). Recent contributions by Kroner and Ng (1998), Engle and Sheppard (2001), Engle (2002), and Tse and Tsui (2002) have developed GARCH models with time-varying covariances or correlations. As an alternative approach, Ramchand and Susmel (1998) and Ang and Bekaert (2002) have estimated a multivariate Markov-switching model and tested the hypothesis of a constant international conditional correlation between stock markets. They obtained that correlation is generally higher in the high-volatility regime than in the low-volatility regime.

In this context, an important issue is how dependency between stock markets can be measured when returns are non-normal. In the GARCH framework, some recent papers have focused on multivariate distributions which allow for asymmetry as well as fat tails. For instance, multivariate skewed distributions, and in particular the skewed Student-t distribution, have been studied by Sahu et al. (2001) and Bauwens and Laurent (2002). In addition, in the Markov-switching context, Chesnay and Jondeau (2001) have tested for a constant correlation between stock returns, while allowing for Student-t innovations.[1] For most types of univariate distributions, however, it is simply impossible to specify a multivariate extension that would allow the dependency structure to be captured. In this paper, we present a new methodology to measure conditional dependency in a GARCH context. Our methodology builds on so-called ''copula'' functions. These functions provide an interesting tool to model a multivariate distribution when only marginal distributions are known. Such an approach is, thus, particularly useful in situations where multivariate normality does not hold. An additional interesting feature of copulas is the ease with which the associated dependency parameter can be conditioned and rendered time varying, even when complicated marginal dynamics are estimated.

We use this methodology to investigate the impact of certain joint stock return realizations on the subsequent dependency of international markets. Many univariate models have been proposed to specify the dynamics of returns. However, given the focus of this work, we draw on recent advances in the modeling of conditional returns that allow second, third, and fourth moments to vary over time. Our univariate model builds on Hansen's (1994) seminal paper. In that paper, a so-called skewed Student-t distribution is derived. This distribution allows for a control of asymmetry and fat-tailedness. By rendering these characteristics conditional, it is possible to obtain time-varying higher moments.[2] This model, therefore, extends Engle's (1982) ARCH and Bollerslev's (1986) GARCH models. In an extension to Hansen (1994),

[1] Some papers also considered how correlation varies when stock market indices are simultaneously affected by very large (positive or negative) fluctuations. Longin and Solnik (2001), using extreme value theory, found that dependency increases more during downside movements than during upside movements. Poon et al. (2004) adopted an alternative statistical framework to test conditional dependency between extreme returns and showed that such a tail dependency may have been overstated once the time-variability of volatility is accounted for.

[2] Higher moments refer to the standardized third and fourth central moments.