Int. J. Production Economics 135 (2012) 702–707

Contents lists available at SciVerse ScienceDirect

Int. J. Production Economics

journal homepage: www.elsevier.com/locate/ijpe

A multi-item fuzzy economic production quantity problem with a finite production rate

Kaj-Mikael Björk [a,b,*]

[a] Åbo Akademi University, IAMSR, IT Department, Finland
[b] Arcada University of Applied Sciences, Department of Business, Information Technology and Media, Finland

ARTICLE INFO

Article history:
Received 9 July 2008
Accepted 8 October 2011
Available online 17 October 2011

Keywords:
Inventory management
EPQ-model
Production-inventory optimization
Supply chain management

ABSTRACT

Inventory Management and production planning are essential tasks for every company in the industry. Therefore, the development of a large set of Economic Order Quantity (EOQ) models is needed. In this paper, a fuzzy multi-item Economic Production Quantity (EPQ) model is developed. This paper contributes to the state-of-the-art with a theoretical study of a problem, where a company has to decide the size of some production batches under uncertain cycle times. The uncertainty will be handled with triangular fuzzy numbers and an analytical solution will be found to the optimization problem.

1. Introduction

In the early 20th century, the first models for the combined optimization of the batch-production and inventory level problem were derived from the basic Economic Order Quantity (EOQ) model. Before this, mathematical methods had started emerging to optimize the size of inventory and orders (the EOQ-model in Harris, 1913) and since then, there has been an increasing number of contributions, which have improved and extended the basic model in many ways. One of these extensions allows for a finite production rate. The different EOQ-models are most often used in a continuous-review setting and it is assumed that the inventory can be monitored every moment in time. It is imperative to acknowledge the importance of production aspects in supply chain management, especially in process-based industries. For these applications, it is important to find solutions that allow the production to be efficient while keeping inventory low (Björk and Carlsson, 2007). This tradeoff problem is found in many supply chains. A specific application that inspired the author to conduct this research is found in the paper industry supply chains in the Nordic countries. These supply chains consist of a few, large paper producing companies and quite many distributors that operate independently from the producers. Typically a large paper machine is producing several products in large quantities. There are often substantial uncertainties found in these supply chains and these cannot be captured by probabilistic measures, cf. Björk and Carlsson (2005) and Carlsson and Fuller (1999). The cycle time (or equivalently the production batch sizes) is often not exact. There are substantial uncertainties in the market conditions (there has been overcapacity on the European fine paper market during the past years, Björk and Carlsson, 2007). This may result in a situation, where management wants to increase batch sizes in order to produce to stock. There are also other reasons for the uncertainty in the batch sizes. All these uncertainties will lead to uncertain cycle time. Therefore, an EPQ-model that models a production process, with several different products, is developed in this paper.

Many EOQ-models and EPQ-models are solved analytically with the use of derivatives. This was also done originally by Harris (1913). There are, however, other methods in use today. Grubbström and Erdem (1999) could prove the EOQ-case with backorders without using derivatives. Mondal and Maiti (2002), on the other hand, used a genetic algorithm to numerically solve a multi-item fuzzy EOQ model. Sometimes uncertainties in the EOQ-models can be modeled stochastically (as done in Liberatore, 1979), but quite often, they cannot be captured with probabilistic means, but only using expert opinions from the companies. This is typically the case with new products, and products with very large seasonal and other unknown variations. For these kinds of uncertainties it is possible to use fuzzy numbers instead of probabilistic approaches (Zadeh, 1965, 1973). It seems that fuzzy set theory can solve inventory problems in a more accurate manner compared to traditional approaches (Guiffrida, 2009). This is also the case for production planning problems (Guiffrida and Nagi, 1998).

There are many contributions within this field, for instance, Chang (2004), who worked out fuzzy modifications of the model of Salameh and Jaber (2000), which took the defective rate of the

* Correspondence address: Åbo Akademi University, IAMSR, IT Department, Finland.
E-mail address: kaj-mikael.bjork@abo.fi

0925-5273/$ - see front matter © 2011 Elsevier B.V. All rights reserved.
doi:10.1016/j.ijpe.2011.10.003

goods into account. There has also been a continuation of this track of research: Jaber et al. (2009), where the entropy costs were included, Khan et al. (2010), where the learning aspect of the inspection of quality was taken explicitly into consideration and Khan et al. (2011b), where inspection errors (as well as imperfect items) also were modeled. A good review over the subject is found in Khan et al. (2011a). Other contributions that assume perfect items are Chang et al. (1998) who numerically solved an EOQ with fuzzy backorder quantities and Yao and Lee (1996) solved it with a fuzzy order quantity. Ouyang and Wu (1998) and Ouyang and Yao (2002) solved an EOQ-model with the lead times as decision variables in addition to the order quantities. Yao et al. (2003) introduced an EOQ-model for two substitutable products with no backorders. Yao and Chiang (2003) used the signed distance method for a fuzzy demand EOQ-model without backorders.

A piece of research closely related to this paper is found in Lee and Yao (1998), where an EPQ-model for single-items was worked out. In addition, the demand and the production quantity were allowed to be triangular fuzzy numbers. They used, however, a numerical optimization method to solve the problem. Islam and Roy (2007) formulated a multi-item EPQ model under space constraints, where the cost parameters were allowed to be fuzzy. Mondal and Maiti (2002) also allowed the parameters to be fuzzy (but not the cycle time) and used a genetic algorithm to numerically solve a multi-item fuzzy EOQ model. This paper addresses a somewhat similar multi-item EPQ-model, where the demand is crisp but the cycle time is kept fuzzy, and as a positive trade-off, an analytical solution is found to the optimization problem. The analytical solution can be found under the assumption of symmetrical triangular fuzzy numbers (describing the cycle time) and with a defuzzification of the objective function before the optimization process begins. The cycle time is also assumed to be the same for every product and the production line is assumed to be shared for all products. The result in this paper is a generalization of the result of Björk (2008), where a similar method was used to find the analytical solution to the fuzzy cycle time, but single-item, EPQ-problem.

The paper is organized as follows: first the crisp model is presented. Then the fuzzy model will be presented and defuzzified in a similar manner to Chang (2004). The defuzzification will be performed with the signed distance method so that the analytical solution can be obtained from the first order derivative (since the objective function is proven to be convex). Finally a small example is given and the paper is concluded with a discussion.

2. The crisp multi-item EPQ model with a finite production rate

The classical multi-item EPQ problem formulation consists of a set of decision variables, i.e. the sizes of the production batches. These variables can be exchanged to the corresponding maximum amount of inventory there will be (directly after the production has stopped, cf. variable q_i in Fig. 1), or the cycle time. In this paper, it is assumed that different products have the same common cycle time. Under a case with no uncertainty, the inventory will undergo a saw teeth behavior, cf. Fig. 1 for an example with two products, for which the production sizes, y_1 and y_2 are equal.

The parameters and variables (that can be assumed to be strictly greater than zero) in the classical multi-item EPQ model with shared cycle time are the following (where the index $i \in I = \{1,2,\ldots,|I|\}$ is denoting the products):

y_i is the production batch size (variable)
K_i is the fixed cost per production batch (parameter)
D_i is the annual demand of the product (parameter)

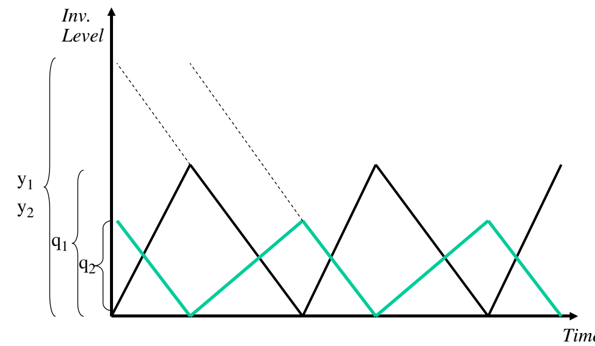

Fig. 1. Representation of a two-product EPQ model with a finite production rate.

Q_i is the maximum difference in the inventory level (variable)
R_i is the annual production rate (parameter)
h_i is the unit holding cost per year (parameter)
T is the cycle time (variable)
q_i is the maximal inventory level (variable).

The total cost function, including production setup costs and inventory holding costs for all products, TCU is given by

$$TCU(y_i,q_i) = \sum_i \frac{K_i D_i}{y_i} + \sum_i \frac{h_i q_i}{2} \tag{1}$$

There is also a need to check that the (shared) production capacity is enough for all products, i.e.

$$\sum_i \frac{D_i}{R_i} \le 1 \tag{2}$$

In addition, the classical EPQ-theory will give the following relationship between the variables y_i, q_i and the cycle time T.

$$q_i = y_i \frac{R_i - D_i}{R_i} \tag{3}$$

The insertion of Eq. (3) into Eq. (1) yields the total cost function to minimize

$$TCU(y_i) = \sum_i \frac{K_i D_i}{y_i} + \sum_i \frac{h_i (R_i - D_i) y_i}{2R_i} \tag{4}$$

The decision variables y_i can also be exchanged with the cycle time T according the formula $T = y_i / D_i$. This will yield the following results:

$$TCU(T) = \sum_i \frac{K_i}{T} + \sum_i \frac{h_i D_i T}{2} - \sum_i \frac{h_i D_i^2 T}{2R_i} \tag{5}$$

Eq. (5) is one version of the crisp (classical) multi-item EPQ-model with shared production capacity and cycle time. This problem can be solved using the derivatives, since all the terms in Eq. (5) are convex. The optimal solution will be a result from

$$\frac{dTCU}{dT} = -\sum_i \frac{K_i}{T^2} + \sum_i \frac{h_i D_i}{2} - \sum_i \frac{h_i D_i^2}{2R_i} = 0 \tag{6}$$

Classical results give us (from Eq. (6)) that the optimal cycle time is

$$T^* = \sqrt{\frac{2\sum_i K_i}{\sum_i h_i D_i - \sum_i (h_i D_i{}^2)/R_i}} \tag{7}$$