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A new analysis of intermittence, scale invariance and characteristic scales applied to the behavior of financial indices near a crash

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Abstract

This work is devoted to the study of the relation between intermittence and scale invariance, and applications to the behavior of financial indices near a crash. We developed a numerical analysis that predicts the *critical date* of a financial index, and we apply the model to the analysis of several financial indices. We were able to obtain optimum values for the *critical date*, corresponding to the most probable date of the crash. We only used data from before the true crash date in order to obtain the predicted critical date. The good numerical results validate the model.

Keywords: Econophysics; Scale invariance; Intermittence; Stock market prices; Financial indices; Crashes

1. Introduction

During the last years the study of log-periodic structures and characteristic scales and the relation with the concept of scale invariance had grown due to the great amount of physical systems presenting log-periodic structures: fluid turbulence [1,2], diamond Ising model [3], earthquakes [4], materials rupture [5], black holes [6] and gravitational collapses [7] among others. In a mathematical context, we recall constructions as the Cantor fractal [3][8], with a discrete scale changes invariant.

The presence of logarithmic periods in physical systems was noted by Novikov in 1966 [9], with the discovery of intermittence effect in turbulent fluids. The relation between both effects has been deeply studied, but it has not been formalized yet.

At the same time, the complexity of international finance has grown enormously with the development of new markets and instruments for transferring risks. This growth in complexity has been accompanied by an expanded role for mathematical models to value derivative securities, and to measure their risks. A new discipline Econophysics, has been developed [10]. This discipline was introduced in 1995, see Stanley et al. [11]. It studies the application of mathematical tools that are usually applied to physical models, to the study of

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financial models. Simultaneously, there has been a growing literature in financial economics analyzing the behavior of major Stock indices [10,12–15].

The Statistical Mechanics theory, like phase transitions and critical phenomena have been applied by many authors to the study of the speculative bubbles preceding a financial crash (see for example [16,17]). In these works the main assumption is the existence of log-periodic oscillation in the data. The scale invariance in the behavior of financial indices near a crash has been studied in Refs. [18–20].

This work is organized as follows: In Section 2 we give a short introduction to the relation between intermittence and scale invariance, the conditions that a function has to satisfy when both effects are present. We analyze the relation with characteristic scales, and we finally present a method that detects characteristic scales in different systems using the previous results. In Section 3 we present a model that predicts the existence of intermittence and characteristic scales in the behavior of a financial index near a crash [20]. In Section 4 we develop the methods that we will use for our numerical analysis.

Finally, we apply the model to the analysis of the behavior of several financial indices: the S&P500 index near the October 1987 crash, and the Argentina MERVAL index as well as the Brazil BOVESPA index and the Mexico MXX index near the October 1997 Asian crash.

2. Scale invariance and intermittence

In this section we analyze the relation between intermittence and scale invariance and we introduce definitions and notation that will be used later. For further details see Ref. [20] and its references.

A function A, that depends on a variable x, is *invariant for the scale change* λx when

$$A(x) = \mu A(\lambda x), \tag{1}$$

where μ is a constant independent of x. For a detailed discussion about this definition see Ref. [20].

Any observable which remains invariant for the scale change $x \to \lambda x$ can be expressed as

$$A(x) = x^{-\log_\lambda \mu} \sum_{n=-\infty}^{\infty} a_n e^{i2\pi n \log_\lambda x}. \tag{2}$$

Now we recall the difference between discrete and continuous invariance. When the observable given by (2) presents *continuous scale invariance*, for any real number λ there exists μ such that condition (1) is fulfilled. We can deduce that in this case $-\log_\lambda \mu$ does not depend on λ, and $a_n = a_0 \delta_{on}$.

When the observable $A(x)$ satisfied Eq. (1) only for numerable values of the λ's, it presents a *discrete scale invariance.*

A particular type of scale invariance is the one arising in the existence of intermittences or "stationary intervals", constant in the logarithm of the independent variable.

The functions that can be obtained from this analysis are

$$f^F(x) = \beta e^{\alpha F(\log_a x)}, \tag{3}$$

$$f^C(x) = \beta e^{\alpha C(\log_a x)}, \tag{4}$$

where β and α are real numbers, α is positive and $F(x) = I(x)$ and $C(x) = I(x) + 1$ are the Floor and Ceiling functions, respectively. Hence, the value obtained when applying the Floor function to a variable x will be the nearest entire number to x from the left, and the value obtained by the Ceiling function will be the nearest entire number to x from the right.

These two functions are discrete scale invariants, and more specifically, they satisfy Eq. (1) only when

$$\lambda = a^n, \quad n \in Z. \tag{5}$$

We recall that the conditions for a function to have discrete scale invariance, after we know that the system has intermittences are the following:

(i) The intermittence intervals must be constant in logarithmic scale, i.e., the steps have to be discrete

$$d(\ln x) = K, \tag{6}$$