Available online at www.sciencedirect.com
ScienceDirect
ELSEVIER
Physica A 377 (2007) 590–598
PHYSICA A
www.elsevier.com/locate/physa

Normalized truncated Levy walks applied to the study of financial indices

M.C. Mariani*, Y. Liu

Department of Mathematical Sciences, New Mexico State University, USA

Received 12 March 2006; received in revised form 20 November 2006
Available online 19 December 2006

Abstract

This work is devoted to the study of the statistical properties of financial indices from developed and emergent markets.

We performed a new analysis of the behavior of several financial indices by using a normalized truncated Levy walk model. We conclude that the truncated Levy distribution describes perfectly the evolution of the financial indices near a crash for both well-developed and emergent markets.

Keywords: Levy flight; Econophysics; Stock market prices; Financial indices

1. Introduction

In recent years a new discipline: econophysics, has been developed [1]. This discipline was introduced in 1995, see Ref. [2]. It studies the application of mathematical tools that are usually applied to physical models, to the study of financial models. Simultaneously, there has been a growing literature in financial economics analyzing the behavior of major stock indices [3–7].

The Statistical Mechanics theory and models, like phase transitions and critical phenomena models, have been applied by many authors to the study of the speculative bubbles preceding a financial crash (see for example Refs. [8,9]). In these works the main assumption is the existence of log-periodic oscillation in the data. The scale invariance in the behavior of financial indices near a crash has been studied in Refs. [10,11].

The statistical properties of the temporal series analyzing the evolution of the different markets have been of great importance in the study of financial indices.

The empirical characterization of stochastic processes usually requires the study of temporal correlations and the determination of asymptotic probability density functions (pdf). The first model describing the option prices evolution is the Brownian motion. This model assumes that the increment in the logarithm of the prices follows a diffusive process with Gaussian distribution [12]. However, the empirical study of the temporal series associated to some of the most important financial indices shows that in short time intervals the associated pdf has greater kurtosis than a Gaussian distribution [6], and that the Brownian motion does not describe

*Corresponding author.
E-mail address: mmariani@nmsu.edu (M.C. Mariani).

0378-4371/$ - see front matter
doi:10.1016/j.physa.2006.11.066

accurately the evolution of financial indices near a crash. The first step in order to explain this behavior was done in 1963 by Mandelbrot [13]. He developed a stochastic model for the evolution of the cotton price by using a stable non-Gaussian Levy process [14]. However, these distributions are not appropriated for working in long-range correlation scales. These problems can be avoided considering that the temporal evolution of financial markets is described by a Truncated Levy Flight (TLF) [15,16].

Most of the studies mentioned before have been done with financial indices of developed markets that have a great volume of transactions. In this work we analyze financial indices corresponding to developed and emergent markets.

Our main interest is to verify that the TLF distribution describes accurately the behavior of financial indices for both developed and emergent markets.

The presentation is organized as follows. In section 2, we give a short introduction to the Levy distributions. In section 3, we present the indices that will be analyzed and the normalized Levy model that will be used for our numerical analysis. Finally, in section 3, we conclude that our results are perfectly compatible with a TLF distribution.

2. The truncated Levy flight

Levy [17] and Khintchine [18] solved the problem of determining the functional form that all the stable distributions must follow. They found that the most general representation is through the characteristic functions $\varphi(q)$ that are defined by the following equation:

$$\ln(\varphi(q)) = \begin{cases} \mathrm{i}\mu q - \gamma|q|^2\left[1 - \mathrm{i}\beta\frac{q}{|q|}\tan\left(\frac{\pi}{2}\alpha\right)\right] & (\alpha \neq 1), \\ \mu q - \gamma|q|\left[1 + \mathrm{i}\beta\frac{q}{|q|}\frac{2}{\pi}\ln|q|\right] & (\alpha = 1), \end{cases}$$

where $0<\alpha\leqslant 2$, γ is a positive scale factor, μ is a real number and β is an asymmetry parameter that takes values in the interval $[-1,1]$. For an extensive discussion see Ref. [1].

The analytic form for a stable Levy distribution is known only in the cases:

$\alpha = 1/2$, $\beta = 1$ (Levy–Smirnov distribution),
$\alpha = 1$, $\beta = 0$ (Lorentz distribution) and
$\alpha = 2$ (Gaussian distribution).

We consider symmetric distributions ($\beta = 0$) with zero mean value ($\mu = 0$). In this case the characteristic function takes the form

$$\varphi(q) = \mathrm{e}^{-\gamma|q|^\alpha}.$$

As the characteristic function of a distribution is its Fourier transform, the stable distribution of index α and scale factor γ is

$$P_L(x) \equiv \frac{1}{\pi}\int_0^\infty \mathrm{e}^{-\gamma|q|^\alpha}\cos(qx)\,\mathrm{d}q.$$

The asymptotic behavior of the distribution for big values of the absolute value of x is given by

$$P_L(|x|) \approx \frac{\gamma\Gamma(1+\alpha)\sin(\pi\alpha/2)}{\pi|x|^{1+\alpha}} \approx |x|^{-(1+\alpha)},$$

and the value at zero $P_L(x=0)$ by

$$P_L(x=0) = \frac{\Gamma(1/\alpha)}{\pi\alpha\gamma^{1/\alpha}}.$$

The fact that the asymptotic behavior for big values of x is a power law has as a consequence that the stable Levy processes have infinite variance. In order to avoid the problems arising in the infinite second moment