Superconsistent estimation and inference in structural econometric models using extreme order statistics[☆]

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Abstract

Data-generating processes whose distributions' supports depend on unknown parameters arise naturally in empirical applications. In such situations the maximum-likelihood estimator is often difficult to calculate and usually has a nonstandard limiting distribution that depends on nuisance parameters. We propose an alternative estimation strategy that is typically simpler to implement than the likelihood approach and allows one to conduct inference using simulation methods. Our proposed estimators are based on the analog estimation principle and bear a striking resemblance to generalized method-of-moments estimators, although here the estimators are generally parameter consistent at rate T rather than the usual rate $\sqrt{T}$. © 2002 Elsevier Science B.V. All rights reserved.

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1. Introduction and motivation

In a wide variety of empirical applications the support of a dependent variable's distribution depends on some or all of the unknown parameters; e.g., the job-search models in Flinn and Heckman (1982) or the procurement-auction models in Paarsch (1992). In such situations the maximum-likelihood (ML) estimator (MLE) is often

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difficult to calculate and usually has a nonstandard limiting distribution, making inference difficult. In particular, the MLE is often a function of extreme order statistics rather than averages, and its limiting distribution is related to the exponential distribution rather than to the normal distribution. Also the distribution of the MLE usually depends on nuisance parameters. In this paper we consider estimation and inference in such models.

First, motivated by difficulties in computing values of and conducting inference for the MLE, we propose estimators based on the analogy principle. These estimators are typically simpler to compute than the MLE and retain some of the advantages that the MLE has over method-of-moments (MM) estimators (MMEs) and generalized method-of-moments (GMM) estimators (GMMEs); viz, a form of parameter superconsistency. [1] Besides being easier to compute the new estimators give rise to statistics for testing parameter restrictions and model specification that have a simple, easily simulated form. In order to gauge relative bias and efficiency of the new estimators, we compare their small-sample performance with that of the MLE and some MMEs using Monte Carlo methods.

Next, we compare the performance of our new estimators with that of the MLE and the MMEs in terms of inference reliability. A simulation-based inference procedure is used for the new estimators. While standard $\sqrt{T}$ asymptotic normality arguments give rise to the usual inference methods for the MMEs, inferences for the MLE are conducted using resampling methods proposed in the statistics literature, in particular a parametric bootstrap method; see, for example, Efron (1982) for an introduction to bootstrap methods and Horowitz (2001) for an elaborate discussion of how such methods have been applied in econometrics.

Finally, we demonstrate that our new estimators can be implemented in a straightforward manner by calculating them as well as the MLE and an MME using data from an application, Paarsch (1992). We also implement a proposed test of specification which yields qualitatively similar conclusions to those reported in Paarsch (1992).

To motivate the importance of our research in terms of empirical applications in economics we consider two canonical examples of support problems which arise naturally in the labor-economic and industrial-organization literatures.

1.1. Job-search example

Our first example, which illustrates the dependence of the dependent variable's support on unknown parameters, comes from the job-search literature begun by Flinn and Heckman (1982) and subsequently surveyed by Devine and Kiefer (1991). Here we borrow from Christensen's and Kiefer's (1991) stylized model of job search, essentially a discrete version of the model developed in Flinn and Heckman (1982). In this model unemployed workers are assumed to know the distribution of wages across firms, but they do not know which firm offers which wage. Wage offers Y are assumed to be distributed with known probability density function $h(y)$ supported on $[0, \infty)$.

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[1] The term superconsistent has been used commonly in the time-series literature to describe estimators that converge more quickly than do those based on sample averages.