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Applying a combined fuzzy systems and GARCH model to adaptively forecast stock market volatility

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ABSTRACT

This paper studies volatility forecasting in the financial stock market. In general, stock market volatility is time-varying and exhibits clustering properties. Thus, this paper presents the results of using a fuzzy system method to analyze clustering in generalized autoregressive conditional heteroskedasticity (GARCH) models. It also uses the adaptive method of recursive least-squares (RLS) to forecast stock market volatility.

The fuzzy GARCH model represents a joint estimation method; the membership function parameters together with the GARCH model parameters make this problem of stock market is highly nonlinear and complicated. This study presents an iterative algorithm based on a genetic algorithm (GA) to estimate the parameters of the membership functions and the GARCH models. In this paper, the GA method is employed to identify a global optimal solution with a fast convergence rate in the context of the fuzzy GARCH model estimation problem studied here. Based on simulation results, we determined that both the estimation of in-sample and the forecasting of out-of-sample volatility performance are significantly improved when the GARCH model considers both the clustering effect and the adaptive forecast.

1. Introduction

Recently, there has been growing interest in modeling and forecasting volatility in the financial and economic literatures [1–8]. The seminal paper of Engle [9] proposed the autoregressive conditional heteroskedasticity (ARCH) construct. Bollerslev [10], a student of Engle, employed the autoregressive moving average (ARMA) model to introduce the generalized autoregressive conditional heteroskedasticity (GARCH) model in a 1986 *Journal of Econometrics* article. The GARCH model involves the joint estimation of a conditional mean and conditional variance equation; it is characterized by a fat tail and excess kurtosis. This model is also a weighted average of past squared residuals, but it has declining weights that never completely reach zero. The model provides parsimonious models that are easy to estimate and have proven to be surprisingly successful in predicting conditional variances. For these reasons, GARCH model is regularly used in analyzing the daily returns of stock market data. However, financial assets are easily impacted by both positive and negative information, and these impacts are often asymmetric. The GARCH model does not recognize the transmissions of volatility that come from the input of positive or negative information. Therefore, this model is not appropriate when the market is asymmetric.

To overcome these shortcomings, GARCH has been extended in numerous ways. The GJR-GARCH model was developed by Glosten et al. [11], while the exponential GARCH (EGARCH) was proposed by Nelson [12]. These models suggest that the negative relationship between volatility and stock prices can be understood by the fact that an increase in unexpected volatility will increase expected future volatility, assuming persistence. Some of these effects can be captured by modifications of linear models, while others demand nonlinear approaches. Unfortunately, due to their complexity, nonlinear models are in very limited use today [1].

Considering these difficulties, there is currently demand for more flexible model. In this paper, we propose a fuzzy GARCH model based on fuzzy systems [13–17]. Fuzzy modeling methods are promising techniques for describing complex dynamics and asymmetries in systems. Combining the ease of implementation and convenience of linear models with an ability to capture complex system correlations, we propose that fuzzy models can also be a judicious choice for analyzing both temporal asymmetries and persistence.

In this paper, we combine GARCH models and fuzzy systems to develop a fuzzy GARCH model. We then apply this new model to real-world financial markets. The process of optimizing fuzzy systems and GARCH model parameters is highly complex and

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nonlinear. A GA-based parameter estimation algorithm is proposed to derive the optimal solution for the fuzzy GARCH model.

GA methods are an approach to optimizing machine learning algorithms inspired by the processes of natural selection and genetic evolution [18,19]. A GA applies operators to a population of binary strings that encode the parameter space. A parallel global search technique emulates natural genetic operators such as reproduction, crossover, and mutation. At each generation, the algorithm explores different areas of the parameter space and then directs the search to the region where there is a high probability of finding improved performance. Because GA simultaneously evaluates many points in a parameter space, it is more likely to converge on the optimal solution [18,20,21]. In particular, there is no requirement that the search space is differentiable or continuous, and the algorithm can be iterated several times for each data point. Accordingly, it is a very suitable approach for time-varying functions.

In practice, in-sample parameter estimation schemes commonly assume a certain volatility model structure for financial stocks [2,5]. Hence, it is necessary to use an adaptive parameter estimation scheme applicable to general out-of-sample volatility models. In this paper, we use the out-of-sample forecast volatility of financial stocks with RLS as an adaptive algorithm in order to track changes in volatility.

The rest of this paper is organized as follows. The next section describes the fuzzy GARCH model. Section 3 presents details on the adaptive forecasting algorithm. Experimental results illustrating the effectiveness of the proposed method are provided in Section 4. The final section of the paper concludes.

2. Fuzzy GARCH model

2.1. GARCH model

Consider the GARCH(p,q) model of daily returns given by [10]

$$y(t) = \sigma(t)\varepsilon(t)$$
$$\sigma^2(t) = \omega + \sum_{i=1}^{q}\alpha_i y^2(t-i) + \sum_{j=1}^{p}\beta_j\sigma^2(t-j), \tag{1}$$

where $y(t)$ is a certain stock market data, $\varepsilon(t)$ is a zero-mean and unit-variance white noise random process, $\sigma^2(t)$ is the conditional variance of $\varepsilon(t)$, t is the time index, and ω, α, and β are unknown parameters to be estimated. Without loss of generality [10], we assume

$$\begin{aligned} &\omega > 0, \alpha_i \geq 0; \quad i = 1, 2, \ldots, q; \qquad q > 0 \\ &\beta_j \geq 0; \quad j = 1, 2, \ldots, p; \qquad p > 0 \\ &\sum_{i=1}^{q}\alpha_i + \sum_{j=1}^{p}\beta_j < 1. \end{aligned} \tag{2}$$

In general, the GARCH(p,q) model can produce similar exact results for different financial datasets. The empirical and theoretical appeal of this model is its simplicity as well as its ability to capture the persistence of volatility. However, the model imposes a symmetrical influence of lagged residuals on current volatility, thereby failing to recognize the volatility caused by a rising or falling stock market. Ignoring this fact can lead to poor prognostic capacities.

To consider the differential effects of the propagation of volatility caused by a rising or falling stock market, we apply fuzzy systems in order to propose a new Fuzzy–GARCH model. The fuzzy GARCH model is described by IF–THEN rules and is employed to ensure that the GARCH(p,q) model can appropriately simulate the fluctuations of the stock market.

2.2. The fuzzy GARCH model

Fuzzy logic systems are universal approximations that can uniformly estimate nonlinear continuous functions with arbitrary accuracy [13,14]. The fuzzy model is a piecewise interpolation of several models that operates using membership functions. In this paper, the fuzzy model is described by IF–THEN rules and employed to ensure that the GARCH(p,q) model can appropriately address the cluster problem. The kth rule of the fuzzy system for GARCH(p,q) is described by

Rule$^{(k)}$:

IF $x_1(t)$ is F_{k1} and $\cdots$ and $x_n(t)$ is F_{kn}, THEN

$$y(t) = \sigma(t)\varepsilon(t)$$
$$\sigma^2(t) = \omega_k + \sum_{i=1}^{q}\alpha_{ki}y^2(t-i) + \sum_{j=1}^{p}\beta_{kj}\sigma^2(t-j) \tag{3}$$
$$\text{for} \quad k = 1, 2, \ldots, L,$$

where $y(t)$ is the output of the system, F_{ki} for $i = 1, 2, \ldots, n$ is the fuzzy set, L is the number of IF–THEN rules, and $x_1(t), x_2(t), \ldots, x_n(t)$ is the premise variables. The premise of a fuzzy implication indicates a fuzzy subspace of the input space, whereas each consequent expresses a local input–output relation in the subspace corresponding to the premise portion of the implication. In this paper, the premise variables include both the previous value of the time series and the previous volatility of the stock market. For example,

$$x_1(t) = y(t-1), \quad x_2(t) = \sigma^2(t-1). \tag{4}$$

As seen in (4), the fuzzy model can successfully capture the phenomena of both leveraging and clustering effects, which result from the sign of the previous time series values and the previous volatility of stock market, respectively.

For the functional fuzzy system, we can use an appropriate operation for representing the premise (e.g., the minimum) [13], and defuzzification may be implemented as follows.

$$\sigma^2(t) = \frac{\sum_{k=1}^{L}u_k(\mathbf{x}(t))\left[\omega_k + \sum_{i=1}^{q}\alpha_{ki}y^2(t-i) + \sum_{j=1}^{p}\beta_{kj}\sigma^2(t-j)\right]}{\sum_{k=1}^{L}u_k(\mathbf{x}(t))}, \tag{5}$$

where

$$\begin{aligned} &u_k(\mathbf{x}(t)) = \prod_{i=1}^{n}F_{ki}(x_i(t)) \\ &\mathbf{x}(t) = [\, x_1(t) \quad x_2(t) \quad \cdots \quad x_n(t)\,], \end{aligned} \tag{6}$$

and $F_{ki}(x_i(t))$ is the grade of membership of $x_i(t)$ in F_{ki}. In this paper, we use the following Gaussian membership function.

$$u_k(\mathbf{x}(t)) = \prod_{i=1}^{n}F_{ki}(x_i(t)) = \prod_{i=1}^{n}\exp\left(-\frac{1}{2}\left(\frac{x_i(t)-c_{ki}}{\xi_{ki}}\right)^2\right), \tag{7}$$

where c_{ki} and ξ_{ki} are the center and spread of the kth rule membership function corresponding to the ith premise variable, respectively. We assume that

$$u_k(\mathbf{x}(t)) \geq 0 \quad \text{for} \quad k = 1, 2, \ldots, L \quad \text{and} \quad \sum_{k=1}^{L}u_k(\mathbf{x}(t)) > 0. \tag{8}$$

Therefore, we obtain

$$h_k(\mathbf{x}(t)) \geq 0 \quad \text{for} \quad k = 1, 2, \ldots, L \quad \text{and} \quad \sum_{k=1}^{L}h_k(\mathbf{x}(t)) = 1, \tag{9}$$

where $h_k(\mathbf{x}(t)) = u_k(\mathbf{x}(t))/\sum_{k=1}^{L}u_k(\mathbf{x}(t))$.