



Nonparametric/semiparametric estimation and testing of econometric models with data dependent smoothing parameters[☆]

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ABSTRACT

We consider nonparametric/semiparametric estimation and testing of econometric models with data dependent smoothing parameters. Most of the existing works on asymptotic distributions of a nonparametric/semiparametric estimator or a test statistic are based on some deterministic smoothing parameters, while in practice it is important to use data-driven methods to select the smoothing parameters. In this paper we give a simple sufficient condition that can be used to establish the first order asymptotic equivalence of a nonparametric estimator or a test statistic with stochastic smoothing parameters to those using deterministic smoothing parameters. We also allow for general weakly dependent data.

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1. Introduction

There is a rich literature on using nonparametric techniques to estimate and test for statistical/econometric models. It is well known that the selection of smoothing parameters is of crucial importance in nonparametric estimation and testing. Various data-driven methods have been proposed in the literature. The least squares cross-validation method is one of the most popular approach used by applied researchers. For density estimation with the kernel method, Stone (1984) has established the optimality result for the least squares cross-validation method under quite weak regularity conditions. Li (1987) provides general optimality results for various data-driven methods with nonparametric series and k -nearest neighbor estimation methods. The optimality results of the cross-validation method in the regression model framework have been studied by Härdle and Marron (1985) and Härdle et al. (1988, 1992). Recently, Hall et al. (2004, 2007) have shown that the least squares cross-validation method has the additional advantage of being able to remove irrelevant covariates in nonparametric kernel conditional density and regression function estimations. However, the literature of nonparametric estimation and testing mainly focus on using deterministic (non-stochastic) smoothing parameters

when working with the asymptotic distribution of a nonparametric/semiparametric estimator or of a test statistic.

Ichimura (2000) considers the asymptotic distribution of nonparametric/semiparametric estimators with data dependent smoothing parameters. Ichimura uses results developed in Pollard (1984, 1990) and Sherman (1994a,b) to establish stochastic equicontinuity conditions for some general U-processes. His approach is technically quite involved, and the sufficient conditions given in Ichimura can be difficult to check for a specific nonparametric estimator or a test statistic. In this paper we suggest a simple condition (from Billingsley (1999), see also Mammen (1992)) to establish stochastic equicontinuity of a general stochastic process. The proof of our approach only involves some low-order moment calculation of a stochastic process indexed by a set parameters (taking values in a bounded set). Therefore, in practice it is much easier to check the conditions given in our paper than those given in Ichimura. Moreover, we allow for general weakly dependent data while Ichimura only considers the independent data case. We demonstrate the usefulness of the method by verifying the conditions for several commonly used nonparametric/semiparametric estimators and test statistics. We show that, under some mild conditions, the asymptotic distributions of these estimators/statistics remain unchanged when one replaces the deterministic smoothing parameters by the stochastic counterparts.

The paper is organized as follows. In Section 2 we consider a simple univariate density estimation problem to motivate and illustrate our general approach of dealing with dependent smoothing parameters in nonparametric estimation. The main results are presented in Section 3. Some nonparametric estimation and testing examples are given in Section 4. Section 5 presents our Monte Carlo

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simulations. Section 6 concludes the paper. The Appendix provides the omitted proofs of Section 3.

2. Description of the approach

Example 2.1 (*Density Estimation: A Motivating Example*). As in Ichimura (2000), we use the simple univariate kernel density estimator as a motivating example to describe our approach. Let X_1, \dots, X_n be a strictly stationary ρ , or β , or α mixing data set (which of course includes i.i.d data as a special case) with density function $f(x)$ ($x \in R$), let

$$\hat{f}(x, h) = \frac{1}{nh} \sum_{t=1}^n k\left(\frac{x_t - x}{h}\right)$$

be the kernel estimator of $f(x)$, where $k(\cdot)$ is a non-negative, bounded symmetric second order kernel function, and h is the smoothing parameter.

Let $h^0 = h_n^0$ be a deterministic sequence that converges to zero. For example, if we choose $h^0 = b^0 n^{-1/5}$ for some positive constant b^0 , it is well established that

$$\hat{\Delta}(x, h^0) \stackrel{\text{def}}{=} (nh^0)^{1/2} [\hat{f}(x, h^0) - f(x) - B(x)(h^0)^2] \rightarrow N(0, V(x))$$

in distribution, (2.1)

where $B(x) = \frac{\kappa_2}{2} \frac{d^2 f(x)}{dx^2}$, $V(x) = \kappa f(x)$, $\kappa_2 = \int k(v)v^2 dv$, and $\kappa = \int k(v)^2 dv$.

Define $m(x, h) = E[\hat{f}(x, h)] - f(x)$, which is the bias term in estimating $f(x)$. Usually, (2.1) is established by first decomposing it into variance and bias terms defined by

$$\begin{aligned} \hat{\Delta}_1(x, h) &= (nh)^{1/2} [\hat{f}(x, h) - E\hat{f}(x, h)], \\ \hat{\Delta}_2(x, h) &= (nh)^{1/2} [m(x, h) - B(x)h^2]. \end{aligned}$$

(2.2)

Then we have

$$\hat{\Delta}(x, h^0) = \hat{\Delta}_1(x, h^0) + \hat{\Delta}_2(x, h^0). \tag{2.3}$$

The bias term is relatively easy to handle since it is non-stochastic. It is straightforward to show that $\hat{\Delta}_2(x, h^0) = o(1)$ as follows.

$$\begin{aligned} \hat{\Delta}_2(x, h^0) &= (nh^0)^{1/2} [m(x, h^0) - B(x)(h^0)^2] \\ &= O((nh^0)^{1/2} (h^0)^3) = o(1), \end{aligned}$$

(2.4)

because $m(x, h^0) - B(x)(h^0)^2 = E[\hat{f}(x, h^0)] - f(x) - B(x)(h^0)^2 = O((h^0)^3)$ by a simple Taylor expansion argument provided that $f(x)$ is three-times differentiable and that its third order derivative is continuous and bounded at an interval containing x as an interior point. The $o(1)$ result follows from the fact that $h^0 \sim n^{-1/5}$.

$\hat{\Delta}_1(x, h^0)$ has zero mean and an asymptotic variance $V(x)$. By utilizing Lyapounov’s Central Limit Theorem, one can show that

$$\hat{\Delta}_1(x, h^0) \rightarrow N(0, V(x)) \text{ in distribution.} \tag{2.5}$$

The asymptotic normality of $\hat{\Delta}(x, h^0)$ stated in (2.1) follows directly from (2.4) and (2.5).

Next, consider a stochastic smoothing parameter \hat{h} with $\hat{h}/h^0 \xrightarrow{P} 1$. This would be the case if \hat{h} is the least squares cross-validation selected smoothing parameter which minimizes a sample analog of $\int [\hat{f}(x, \hat{h}) - f(x)]^2 dx$, and h^0 is the deterministic smoothing parameter that minimizes the leading term of the integrated mean squares error: $\int E[\hat{f}(x, h) - f(x)]^2 dx$. Then under the assumption that $f(x)$ has a non-vanishing second derivative function, we know that $h^0 = b^0 n^{-1/5}$ for some finite positive constant b^0 , and that $\hat{h}/h^0 - 1 = o_p(1)$ (e.g., Stone (1984)).

We want to establish the asymptotic distribution of $\hat{f}(x, \hat{h})$. If one can show that $\hat{\Delta}(x, \hat{h}) - \hat{\Delta}(x, h^0) = o_p(1)$, then $\hat{\Delta}(x, \hat{h})$ and $\hat{\Delta}(x, h^0)$ have the same asymptotic distribution, and by (2.1) we know that $\hat{\Delta}(x, \hat{h}) \xrightarrow{d} N(0, V(x))$.

To show that $\hat{\Delta}(x, \hat{h}) - \hat{\Delta}(x, h^0) = o_p(1)$, we use (2.3) to decompose $\hat{\Delta}(x, \hat{h}) - \hat{\Delta}(x, h^0)$ into two parts (with $h^0 = b^0 n^{-1/5}$ and $\hat{h}/h^0 \xrightarrow{P} 1$).

$$\begin{aligned} \hat{\Delta}(x, \hat{h}) - \hat{\Delta}(x, h^0) &= [\hat{\Delta}_1(x, \hat{h}) - \hat{\Delta}_1(x, h^0)] \\ &\quad + [\hat{\Delta}_2(x, \hat{h}) - \hat{\Delta}_2(x, h^0)]. \end{aligned}$$

(2.6)

Asymptotic equivalence between $\hat{\Delta}(x, \hat{h})$ and $\hat{\Delta}(x, h^0)$ follows if one can show that $\hat{\Delta}_l(x, \hat{h}) - \hat{\Delta}_l(x, h^0) = o_p(1)$ for $l = 1, 2$.

We write $\hat{h} = \hat{b}n^{-1/5}$, then $\hat{h}/h^0 \xrightarrow{P} 1$ is equivalent to $\hat{b}/b^0 \xrightarrow{P} 1$. Define $B = [B_1, B_2]$ with $0 < B_1 < b^0 < B_2 < \infty$ (B is a bounded set that contains b^0 as an interior point), and

$$I_{n,1}(b) = \hat{\Delta}_1(x, h)|_{h=bn^{-1/5}} \text{ with } b \in B. \tag{2.7}$$

$I_{n,1}(b)$ is a stochastic process indexed by $b \in B$. We will show in the next section that a sufficient condition for $\hat{\Delta}_1(x, \hat{h}) - \hat{\Delta}_1(x, h^0) = o_p(1)$ is that

$$E\{[I_{n,1}(b') - I_{n,1}(b)]^2\} \leq C(b' - b)^2 \tag{2.8}$$

for all $b, b' \in B$, where C is a finite positive constant. Note that (2.8) is quite easy to establish as it only involves a second moment calculation of $I_{n,1}(b)$.

Similarly, define

$$I_{n,2}(b) = \hat{\Delta}_2(x, h)|_{h=bn^{-1/5}} \text{ with } b \in B. \tag{2.9}$$

In the Appendix we show that a sufficient condition for $\hat{\Delta}_2(x, \hat{h}) - \hat{\Delta}_2(x, h^0) = o_p(1)$ is that

$$\sup_{b \in B} |I_{n,2}(b)| = o(1). \tag{2.10}$$

Eq. (2.10) is also very easy to verify because $I_{n,2}(b)$ is non-stochastic. Assuming that $f(x)$ has a continuous and bounded third derivative, by Taylor series expansion we have $|I_{n,2}(b)| = |\hat{\Delta}_2(x, h)|_{h=bn^{-1/5}} = O((nh)^{1/2} h^3) = o(1)$ uniformly in $b \in B$ ($h = bn^{-1/5}$, $b \in B$). Hence, (2.10) holds.

Therefore, (2.10) implies that $\Delta_2(x, \hat{h}) = \Delta_2(x, h^0) + o_p(1) = o_p(1)$. In the Appendix we also show that (2.8) holds true provided that $k(\cdot)$ satisfies a Lipchitz condition: $|k(u) - k(v)| \leq \zeta(v)|u - v|$, where $\zeta(v)$ is a bounded function and that $\int \zeta(v)v^2 dv$ is finite. Consequently by (2.6), (2.8) and (2.10), we have $\hat{\Delta}(x, \hat{h}) = \hat{\Delta}(x, h^0) + o_p(1)$. Hence, by (2.1) we know that,

$$\begin{aligned} \hat{\Delta}(x, \hat{h}) &= (n\hat{h})^{1/2} [\hat{f}(x) - f(x) - B(x)\hat{h}^2] \rightarrow N(0, V(x)) \\ &\text{in distribution.} \end{aligned}$$

(2.11)

We now provide the intuition as why one should expect that (2.8) and (2.10) imply $\Delta_l(x, \hat{h}) - \Delta_l(x, h^0) = o_p(1)$ for $l = 1, 2$. By Theorem 13.5 of Billingsley (1999, p.142), or Theorem 15.6 of Billingsley (1968, p.128), (see also Ossiander (1987), and Bai (1994, 1996)), we know that (2.8) implies that the stochastic process $\Delta_1(x, \cdot)$ is tight and stochastically equicontinuous for $b \in B$. That is, for any $\epsilon > 0$,

$$\begin{aligned} \lim_{n \rightarrow \infty} \lim_{\delta \rightarrow 0} \Pr \left[\sup_{b, b' \in B; |b' - b| < \delta} |I_{n,1}(x, b') - I_{n,1}(x, b)| > \epsilon \right] \\ = 0. \end{aligned}$$

(2.12)

Eq. (2.12) implies that $I_{n,1}(\hat{b}) - I_{n,1}(b^0) = o_p(1)$ conditional on $\hat{b} \in B$. Also, the condition that $\hat{b} - b^0 = o_p(1)$ ensures that

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