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American contingent claims under small proportional transaction costs

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Abstract

American options are considered in the binary tree model under small proportional transaction costs. Dynamic programming type algorithms, which extend the Snell envelope construction, are developed for computing the ask and bid prices (also known as the upper and lower hedging prices) of such options together with the corresponding optimal hedging strategies for the writer and for the seller of the option. Representations of the ask and bid prices of American options in terms risk-neutral expectations of stopped option payoffs are also established in this setting.

Keywords: American options; Transaction costs

1. Introduction

In the presence of small proportional transaction costs in the form of a bid-ask spread of the underlying stock prices we shall consider American options exercised by the physical delivery of a portfolio of cash and stock.

For example, when a call option with strike price K is exercised by physical delivery, the holder of the option pays K and receives 1 share, that is, the option is exercised by the delivery of a portfolio $(-K, 1)$ of cash and stock. If buying or selling stock incurs transaction costs, then physical delivery is not equivalent to cash settlement.

Chalasani and Jha (2001) investigated American options with cash settlement only under (not necessarily small) proportional transaction costs. They obtained general representations involving

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so-called randomised stopping times for the ask and bid prices of American contingent claims. However, no algorithmic procedure for computing the prices was proposed. Indeed, Chalasani and Jha commented that "The computation of the expressions for the upper and lower hedging prices appears non-trivial. It would be useful to design efficient algorithms for approximating the values of these expressions." (Chalasani and Jha, 2001, p. 72).

Here we put forward two algorithms extending the Snell envelope construction, one for computing the ask price (the upper hedging price) and one for the bid price (the lower hedging price) of an arbitrary American contingent claim under small proportional transaction costs in the binary tree model. We also construct optimal hedging strategies for the option writer as well as for the seller, and establish representations of the ask and bid option prices in terms of risk-neutral expectations of stopped payoffs. In contrast to Chalasani and Jha (2001), our results involve ordinary rather than randomised stopping times, and we consider American options with physical delivery rather than ones with cash settlement only.

Numerical examples demonstrating the applicability of the pricing algorithms are provided, and certain novel and interesting features are noted, such as the non-equality of bid prices for American and European calls, or different optimal stopping times for the writer and the holder of an American option under transaction costs.

Other papers devoted to American options under proportional transaction costs include Davis and Zariphopoulou (1995), Levental and Skorohod (1997), Mercurio and Vorst (1997), Kociński (1999, 2001), Perrakis and Lefoll (2000, 2004), Constantinides and Zariphopoulou (2001), Jakubenas et al. (2003), Constantinides and Perrakis (2004) and Bouchard and Temam (2005).

For a wider context we refer to the extensive literature concerned with pricing and hedging European options under proportional transaction costs. This problem was studied by Merton (1990), Dermody and Rockafellar (1991), Boyle and Vorst (1992), Bensaid et al. (1992), Edirsinghe et al. (1993), Jouini and Kallal (1995), Kusuoka (1995), Naik (1995), Shirakawa and Konno (1995), Soner et al. (1995), Cvitanić and Karatzas (1996), Koehl et al. (1996, 1999, 2002), Cvitanić et al. (1999), Levental and Skorohod (1997), Perrakis and Lefoll (1997), Stettner (1997, 2000), Rutkowski (1998), Touzi (1999), Jouini (2000), Ortu (2001), Palmer (2001a,b), Kociński (2004), and many others.

In the majority of these papers the authors assume a stock price process S_t under no-arbitrage conditions, and introduce transaction costs by multiplying S_t by constant factors $1 + \lambda$ and $1 - \mu$ for some $\lambda, \mu \geq 0$. Here we follow the more general approach of Jouini and Kallal (1995) involving bid-ask spreads $S_t^b \leq S_t^a$ for the stock price. As pointed out by Jouini (2000), the spreads can be interpreted as proportional transaction costs in the above sense, but can also be explained by the buying and selling of limit orders. Accordingly, S_t^a and S_t^b can be thought of as the prices ensuring liquidity in the stock market, that is, at which stock can be bought or, respectively, sold on demand. The spreads, therefore, include proportional transaction costs, but are not limited to them.

The lack of arbitrage in a model with bid-ask spreads was characterised by Jouini and Kallal (1995) in terms of the existence of suitably defined risk-neutral measures and associated stock price processes, see Theorem 2.1. We use their results here as our starting point. See also Ortu (2001).

Small proportional transactions costs similar to those considered in the present paper have been studied by a number authors in various contexts, for example, by Bensaid et al. (1992), Koehl et al. (1999), Kociński (1999, 2001), and Melnikov and Petrachenko (2005). The definitions of small transaction costs differ slightly between these various approaches, but there is a substantial overlap, also with that adopted in the present paper.