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The single-period inventory problem: Extension to random yield from the perspective of the supply chain☆

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ABSTRACT

A special form of the single-period inventory problem (newsvendor problem) with a known demand and stochastic supply (yield) is studied. A general analytic solution for two types of yield risks, additive and multiplicative, is described. Numerical examples demonstrate the solutions for special cases of uniform distribution yield risks. An analysis of a two-tier supply chain of customer and producer reveals that the customer may find it optimal to order more than is needed, since a larger order increases the producer's optimal production quantity.

1. Introduction

In this paper a special form of the single-period inventory problem (newsvendor problem) with a known demand and stochastic supply (yield) is explored. Two types of yield risks are considered: additive and multiplicative. The model is then extended for achieving supply chain coordination and integration. In the first part of the paper we consider a special form of the newsvendor problem with a fixed and known demand and a random yield. We analyze the problem under additive and multiplicative yield risks and derive the optimal production quantity. The motivation for exploring the problem is derived from the numerous real world examples where a producer receives a firm order from a buyer for some given quantity, but the production system has a random yield. Production systems with random yield can be found particularly in agriculture (such as for the production of vegetables or eggs), the chemical industry (such as for the production of special chemicals or tailor made chemicals) and the electronics industry (such as for the production of special processors or silicon chips). Specific problems that may arise include how many lettuce plants must a farmer sow to supply a requirement for a given quantity of lettuce; how many tons of raw materials should a chemist use for producing a special chemical to supply a given quantity for his customer; how many silicon chips must Intel® produce in order to supply enough chips to pass a quality test and meet its customer demands? On the one hand, randomness, quality problems and nonconformity may reduce quantities of end products, while on the other hand, overproduction may increase production costs, holding costs and even cause high destruction costs in the case of dangerous chemicals that are not needed.

In the second part, we extend the newsvendor model to include the distributor problem. The distributor, who is situated in the middle of the supply chain between the producer and the end user, has private information about the demand of the customer (the end user). The distributor's problem involves determining the optimal quantity that should be ordered with the aim of urging the producer to produce the appropriate quantity that the end user needs under random yield. The purchase of goods by a distributor (or retailer) from a producer which is then resold to some end user is a common phenomenon in various businesses. Wal-Mart

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Stores, Inc., for example, a large retailer with many suppliers, like many other large retailers, has significant market power over its suppliers due to its size and direct access to end users. Therefore, the formulation of a model with extra power to a distributor over his producers can be justified.

We formulate the distributor's problem of finding the optimal order quantity as a mathematical programming problem, where the producer's problem of finding the optimal production quantity is a constraint in the distributor's mathematical programming problem. Although the distributor can calibrate the producer's production decision by various means (buyback price, coordination contract, etc.), we believe that setting the order size as a coordination mechanism is a preferred tool. The main advantage is that the distributor does not have to negotiate frequently with the producer about prices, quantity discounts, buyback, risk sharing or other parameters, which can be left to the determination of market forces. Nowadays dynamic markets, prices and other parameters can change rapidly due to external forces. Consequently, it is clear to supply chain members that as a result of external demands, order quantities can change from time to time due to customers' needs and demands. Therefore, frequent changes in order quantities would not be a cause for renegotiation, unless the supply chain members look for a long-term commitment. Long-term commitment in present day dynamic markets with many new opportunities for supply are usually not desired. Our solution for supplier–buyer coordination without any special agreement beyond a common order size under regular market conditions avoids these obstacles. Moreover, our model can be modified to take into consideration possibilities of changes in buyback price, purchasing price or other parameters which may improve the consolidated profit (distributor+producer) by more tightened supply chain integration.

The results of the paper can be applied in two ways. One is for a producer who has a single-period inventory problem with a known demand and stochastic yield to solve the problem by setting the optimal production quantity from his/her point of view. The other way is for a distributor to use his/her power for channel coordination by calculating the optimal quantity to be ordered from the producer to maximize profits. It is interesting to note that under some conditions, it is optimal for the distributor to order more than the customer needs. When that happens, a larger order is better for the producer and may even be to the end-users' advantage as well since it increases the probability of full supply.

2. Literature review

The literature is surveyed from two aspects: (1) the single-period inventory problem with known demand and random yield and (2) supply chain coordination and integration.

2.1. Single-period inventory problem with known demand and random yield

A variety of extensions to the single-period inventory problem (the newsvendor problem) and other newsvendor type problems have been introduced since the problem was first presented by Within [1]. Yano and Lee [2] presented a thorough review of single stage single-period models that dealt with lot sizing models with random yields. Khouja [3] classified extensions to the basic newsvendor problem into 11 categories including random yields, which is a main issue in the present paper. Khouja focused on the literature which deals with the single-period inventory problem extensions in which the quantity received is a random variable. The random yields considered in that literature were due to random proportions of defective units or random production capacity and outputs.

Karlin [4] assumed that the number of good units in a lot is a random variable with a known probability distribution. He limited the ordering decisions to two alternatives: do not order or order from a choice of set levels, which does not allow for a range of order sizes.

Shih [5] assumed that defective units should be returned to the manufacturer at his own expense. He also assumed that the percentage of defectives is a random variable with a known probability distribution, and derived the expected cost function, provided proof of its convexity and derived the necessary optimality condition for the amount received for any distribution of quantity demanded and the percentage of defectives.

Noori and Keller [6] considered a single-period inventory model with stochastic demand when the amount received is a random variable. They obtained analytical results for the amount received for uniformly and exponentially distributed demand.

Ehrhardt and Taube [7] generalized Shih's [5] model by dealing with general forms of holding and shortage costs instead of the linear case and derived the necessary optimality conditions. For uniformly distributed demand, these authors provided a closed-form expression for the amount received.

Gerchak et al. [8] also dealt with random yield and assumed that there is some existing initial stock, I. They allowed the cost to be proportional to the planned amount Q or to the net yield, and showed that the expected cost is concave in I and Q. Furthermore, they found a critical level of I, above which no order will be placed under a certain yield, and that this level is the same under random yield. Finally, they showed that, unlike certain yield, the optimal policy under random yield is not of the "order-up-to" type.

Henig and Gerchak [9] did not assume that the yield constitutes a proper fraction of Q, which makes the model applicable to situations where the input level and yield size are not measured in the same units. Instead, they assumed that the production costs depend on the realized yield and then generalized their results to where the production costs depend on both Q and the realized yield.

Parlar and Wang [10] analyzed a situation in which the newsvendor uses two suppliers, each having random yield. These authors assumed that the suppliers have different yield distributions and prices, and used the stochastically proportional yield assumption. They concluded that diversification may be useful since it can reduce the overall yield variability.

Ciarallo et al. [11] analyzed a problem in which uncertainty is a result of random capacity rather than yield. They