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# Stability of inventory dynamics in supply chains with three delays

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## ABSTRACT

Distinctive delays accounting for lead time in manufacturing, transportation of products and decision-making are considered in a single link supply chain that is modeled by continuous-time differential equations via system thinking. As seen in the literature, behavior of inventory variations in the presence of delays can become undesirably oscillatory. The novelty of this work is the analytical characterization of these oscillations with respect to two intrinsic parameters of the supply chain and the three delays considered separately. Particularly, inventory dynamics is characterized on the so-called stability maps drawn in the space of three delays, displaying which combinations of each delay lead to (un)desirable inventory behavior. Arising from the characterization is also a novel ordering policy design with which the inventory variations can be rendered *insensitive* to detrimental effects of delays. Case studies are provided along with managerial interpretations.

## 1. Introduction

Inventory dynamics exhibit quite complex behavior in supply chains (SC) since inventory level variations are the end results of combined decision making, manufacturing and product shipment activities which are dynamically adapted against unpredictable and sometimes artificial consumer demand. While surfeit of inventories (overshoot) cause increased stocking costs, deficit of inventory levels (undershoot) may increase freight costs and the risk of depletion of inventories, all of which indicate inefficiency. Consequently, cost effective supply chain management naturally requires thorough understanding of decision making, manufacturing and product shipment dynamics that directly affect the underlying mechanisms of inventory behavior.

One of the most critical parameters in supply chain management (SCM) is the delay (Sarimveis et al., 2008; Riddalls and Bennett, 2002b; Sterman, 2000). Delay is inevitable in SC due to physical constraints related to lead times (in manufacturing), transportation and delivery times (shipments), decision making durations (human behavior) and information availability (communication delays, data collection delays). In the presence of delays, what is known to the SC manager is not what is happening in the chain, but it represents the information regarding the SCs behavior in the time history. Moreover, there are multiple sources of delays in the SC and these delays are quantitatively different (An and Ramachandran, 2005). Therefore, available information pertaining to SC carries multiple delay signatures. What is detrimental to SCM is that delays mislead decision makers. This consequently prevents achieving successful SCM.

Although it is known that delays bring detrimental effects, in some cases it is preferable that managers wait (adding delay) in order to observe the trends in the SC and in the market before making critical decisions (Sterman, 2000). Clearly, it is not straightforward to comment on the effects of delays to SCM. These two counter-intuitive arguments justify the need to study delay effects to dynamic behavior of the SC (Croson et al., 2004; Riddalls et al., 2000; Beamon, 1998; Hafeez et al., 1996).

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We quest if there are ways to uncover the effects of delays to inventories and to SCM. If these effects can be understood with respect to intrinsic parameters defining the SC, then it would be possible to come up with new management strategies that can combat against undesirable effects of delays. This is exactly what forms the main objective here and it is aligned with the earlier work in Sarimveis et al. (2008), Warburton (2004), Ge et al. (2004), Riddalls and Bennett (2002b), Sterman (2000, 1989), and Simchi-Levi et al. (2000). By performing stability analysis of the SC, we wish to reveal various dynamical behaviors of the SC and inventory levels with respect to delays and the parameters pertaining to management strategies. The stability/instability definitions used in this paper are along the lines of for instance Riddalls and Bennett (2002a) and Naim et al. (2004). For various combinations of management strategies, we are particularly interested in finding the delay values with which the inventories behave in a desirable way where inventory perturbations damp out (which we call as "stability") rather than exhibiting oscillatory behavior (which we call as "instability"). It may be true that SC dynamics may eventually stabilize itself with the presence of bounds such as capacity limits, however, the long durations of inventory oscillations, which are known to have large periods, may put the SC into large financial losses before such bounds and extremis may take over and stabilize the SC. In this sense, the contribution of this paper can be seen as the characterization of delay effects to such persistent and undesirable transient behaviors observed in the inventory levels. As a result of our analysis, the SC manager has a decision making tool with which the SC can be operated in a stable regime based on various strategies and delays. With the tools we provide, it is also possible in some cases to dictate desirable inventory behavior by scheduling some of the activities with appropriate delays similar to the work in Lee and Feitzinger (1995), and to choose appropriate ordering policy with which the inventory levels are rendered *insensitive* to undesirable effects of delays. The results of this paper bridge the gap between surfacing undesirable effects of delays in SC and how to make proper decisions to avoid these effects in SCM.

The mathematical framework of the study is constructed on Laplace domain, which is known to have been used first time in 1952 (Simon, 1952) for studying the stability of supply chains by Nobel Prize winner Herbert Alexander Simon. Forrester (1961) also derived differential equations for the same reason. Furthermore, Towill (1982) deployed Laplace transform for studying inventory and order based production control system. In Table 4 of Disney et al. (2006), it was shown that continuous time domain studies are more preferable due to various reasons except one, that is, the *pure delays*. The work presented here removes this concern, making continuous time domain analysis and its connection with Laplace transform a perfect platform to analyze SC and SCM.

The particular SC problem studied in this paper is along the lines of Towill (1982), John et al. (1994), Riddalls and Bennett (2002b, 2003), where we consider an Automatic Pipeline Inventory and Order Based Production Control System (APIOBPCS) with two intrinsic deterministic parameters regulating a single inventory of a single product shipped via a single link transportation path. This model is also used in simplified forms in Hafeez et al. (1996) and Lewis et al. (1995). Interestingly APIOBPCS is similar to the heuristic stock acquisition strategy of Sterman which Sterman (1989) obtained from experiments involving multiple users playing a beer distribution game. What is different in this paper is that delay originates from three dissimilar physical sources hence we consider three different delays. These delays emerge from (i) decision making, (ii) production and (iii) transportation time. Hitherto, effects of each one of the three delays together were not investigated within a unified model, despite the fact that these delays are known to exist, (Ge et al., 2004; Riddalls and Bennett, 2002b; Sterman, 2000). With the analysis performed in this paper, we wish to present a broader picture as to how each delay governs the stability mechanisms of the SC.

The paper is organized as follows. In Section 2, problem formulation is presented including the details of the mathematical model and the consideration of delays. Section 3 develops the system thinking and the authors' earlier work (Sipahi and Delice, 2009) towards revealing the delay effects to inventory regulation and designing ordering policy in the presence of delays. Section 4 presents case studies and managerial decision making strategies extracted by exploiting the tools developed in Section 3. Discussions, limitations and future research directions conclude the paper in Section 5.

Notations are standard. We use $s$ for Laplace variable, $\mathbb{C}$ for complex plane, $j$ for complex number, $j=\sqrt{-1}$. $\Re(s)$ denotes real part of $s$, $\Im(s)$ corresponds to imaginary part of $s$. Complex variable $s$ lies in the left half complex plane $\mathbb{C}_-$ when $\Re(s)<0$, and in the right half complex plane $\mathbb{C}_+$ when $\Re(s)>0$. $\mathbb{R}$, $\mathbb{R}_{+,-}$, $\mathbb{Z}$, $\mathbb{Z}_+^\infty$ stands for the set of real numbers, positive real numbers, integer numbers and all positive integer numbers, respectively. $\mathscr{C}_k$ denotes a circle with unity radius located on the origin of $x_k-y_k$ plane. $\boldsymbol{I}$ is the identity matrix with appropriate dimensions, and inf denotes the greatest lower bound of a set.

## 2. Problem formulation and preliminaries

In this section, the mathematical model of the SC with delays is presented. For the SC model, we follow the earlier work in Towill (1982), John et al. (1994), and Riddalls and Bennett (2002b, 2003) in which a delay accounting for lead time is considered. For representing the delay effects, similar research lines as in Riddalls and Bennett (2002a, b, 2003) are adapted. The number of delays in this paper is, however, three and once the modeling is established in this section, we discuss in the next section how three delays are incorporated and how the stability of SC is analyzed.

### 2.1. Mathematical modeling of delays

In order to realistically create the SC model, we consider lead times and transportation times as pure