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Dynamic inventory rationing for systems with multiple demand classes and general demand processes

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ABSTRACT

We consider the dynamic rationing problem for inventory systems with multiple demand classes and general demand processes. We assume that backorders are allowed. Our aim is to find the threshold values for this dynamic rationing policy. For single period systems, dynamic critical level policy is developed and the detailed cost approximation subject to this policy is derived. For multiperiod systems, a dynamic rationing policy with periodic review is proposed. The numerical study shows that our dynamic critical level policies are close to being optimal for various parameter settings.

1. Introduction

Inventory is an important driver in modern supply chains and has traditionally been used to provide a buffer against demand uncertainty or increased service levels. However, there are costs associated with holding inventory, such as opportunity costs, storage costs, obsolescence costs, insurance costs, and damage costs. Hence, organizations face a trade-off between incurring inventory and servicing their customers. However, inventory can serve purposes beyond its traditional role because heterogeneous customers have different service needs and priorities. This means that firms can make tactical decisions with regard to the rationing of inventory and can set different pricing and service levels according to their customer service needs. By providing a differentiated service according to customer needs, firms can benefit, because this helps to increase market size, and thereby revenue. For example, firms can charge higher prices to customers who need immediate service and can charge less for customers who only need a normal service. This practice is common in many industries, such as the airline industry, online retailing, and the services parts industry. The airline industry usually charges different prices for the same seat, and online retailers, such as Amazon.com, provide expedited and normal shipping services. The services parts industry also charges customers according to services delivery contracts.

For a firm to successfully adopt a different pricing or service level strategy for the same inventory, the main assumption is that customers can be segmented according to their different service needs and priorities. The key challenge is how to allocate the inventory to different segments of customers. For motivation, this paper uses the example of a firm that has an extensive network providing spare parts, which are used to maintain or replace failed equipment parts at the customer's site. It has a major regional distribution center, which serves its customers. Requests for spare parts are prompted by parts failure and by scheduled maintenance. Requests prompted by parts failure must be rectified immediately, whereas those prompted by scheduled maintenance can wait. Hence, in any period, the distribution center may face these two types of demand from its customers. In this situation, a firm may adopt the rationing policy that when inventory is low, only urgent demand for parts is satisfied. The inventory level at which low-priority requests are rejected is sometimes known as the critical, or threshold level. The policy of reserving stock is termed the. Many researchers have explored practical examples of inventory rationing, such as Kleijn and Dekker (1998), Deshpande et al. (2003), and Cardós and Babiloni (2011).

There are two kinds of critical-level policies: stationary and dynamic. For stationary policies, the critical levels are constant. Much research has been carried out on stationary critical level policy. For make-to-stock production systems, the stationary critical level rationing policy is optimal for specific cases (Ha, 1997a, 1997b, 2000; Gayon et al., 2004). For exogenous inventory supply problems, researchers such as Melchiors et al. (2000), Deshpande et al. (2003), and Arslan et al. (2007) propose stationary policies and then determine the optimal parameters for the critical levels that minimize inventory costs. Others such as Nahmias and Demmy (1981), Moon and Kang (1998), Cohen et al. (1988), Dekker et al. (1998), and Möllering and Thonemann (2009) determine the stationary critical levels for inventory systems operating under different service levels.

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For dynamic policies, the critical levels may change over time. Topkis (1968) considers dynamic inventory rationing policy for single period and multiple period systems with zero lead times. A dynamic programming model is proposed in which one period is divided into many small intervals. Topkis also shows that the optimal rationing policy is dynamic. However, he fails to show that the critical level is nonincreasing over time. Evans (1968) and Kaplan (1969) extend results from Topkis (1968) and explore two demand classes. Melchiors (2003) considers dynamic rationing policy under an inventory system with a Poisson demand process and an (s, Q) ordering policy in which backordering is not allowed. Lee and Hersh (1993) consider dynamic rationing policy for an airline seating problem. Teunter and Klein Haneveld (2008) develop a continuous time approach to determining the dynamic rationing policy for two Poisson demand classes under the assumption that there is no more than one outstanding order. However, its computational results are tractable only for limited settings. Fadiloglu and Bulut (2010) propose a heuristic rationing policy called "rationing with exponential replenishment flow" for continuous-review inventory systems. All except Topkis (1968) consider only two demand classes. However, the limitation of his approach is that the state spaces grow exponentially large when the number of demand classes increases. Even for two demand classes, the state space can be very large. Moreover, many researchers assume a Poisson distribution.

In this paper, we develop an approximation approach to deriving the dynamic threshold level for inventory systems with multiple demand classes and general demand processes. This approximation approach is based on comparing the marginal costs of accepting and rejecting a demand class when it arrives. It is also assumed that when this demand class is rejected, all future demands from this class will be rejected until the next replenishment arrives. Unlike existing work, this method can deal with general demand processes, and is efficient in solving problems with more than two demand classes. To illustrate the effectiveness of the proposed policy, we conduct numerical analysis. The results show that the proposed policies are close to being optimal under various parameter settings when demand follows a Poisson process. The Poisson process is used because we want to compare our solutions with the optimal solution.

Our paper is organized as follows. In Section 2, we consider a single period system with multiple demand classes. We derive the dynamic critical levels based on the concept of marginal cost. In Section 3, we consider multiperiod systems with periodic review policy. The rational policy in Section 2 is extended for multiperiod systems. In Section 4, numerical studies are conducted to investigate the performance of the proposed approaches. In Section 5, we summarize the results and discuss some possible extensions.

2. Inventory rationing for single period systems

In this section, the inventory rationing problem for a single period system is considered. The goal is to find a good dynamic rationing policy. We first examine the dynamic critical levels for only two demand classes and then extend the results for a general number of demand classes. Then, we develop an approximation method for computing the expected total costs associated with our rationing policy.

2.1. Model formulation

Consider a single period inventory system with a period length of u. There is a single product with demands from K different customer classes. We assume that the demand processes of all customer classes are independent and stationary and that these demands can be partially satisfied. We let $t=u$ denote the beginning of the period, we let $t=0$ denote the end of the period, and we let $X(t)$ denote the on-hand inventory at the remaining time $t \in [0,u]$. For each unit stored, we assume a holding charge of h per unit of time. At the beginning of the period, we assume that the initial on-hand inventory is given, and is equal to x (i.e., $X(u)=x$). During the period, each customer demand may either be satisfied or rejected according to our rationing policy. The rejected demand is backordered and a backorder cost is applied. We define the backorder cost for class i as $\pi_i+\hat{\pi}_i t$, where $t \in [0,u]$ is the remaining time to the end of the period. Note that π_i is a fixed penalty cost to reject a demand from class i, which may represent the loss of customers' loyalty. Moreover, $\hat{\pi}_i$ is the per-unit-of-time cost to hold a demand from class i to the end of the period without loss of goodwill or the order. Without loss of generality, we arrange demand classes in the order of nonincreasing backorder cost. That is, $\pi_i \geq \pi_j$ and $\hat{\pi}_i \geq \hat{\pi}_j$ for demand classes $i<j$.

At the end of the period ($t=0$), we assume that all backorders must be fulfilled. We propose using the remaining inventory to fulfill these backorders first and if this is insufficient, we propose the purchase of additional units to fulfill the remaining backorders from the open market. If the remaining inventory exceeds the backorders, the surplus is sold at salvage value on the same market. We assume that both the salvage value and the additional purchasing cost on the open market equal c_0 per unit of product at the end of the period.

To determine the dynamic rationing policy, we need to compute critical levels over time. Define $s_i(t)$ to be the dynamic critical level of class i for the remaining time $t \in [0,u]$. When $X(t) > s_i(t)$, we satisfy the demand from class i. Otherwise, the demand from class i is rejected. As we have arranged demand classes in the order of nonincreasing backorder cost, we have $s_i(t) \leq s_j(t)$ for classes $i<j$. We also define $H(t,X(t))$ as the expected cost for the remaining time t. Thus, our objective is to find the rationing policy that minimizes the expected total cost, which can be written as follows:

$$\min_{\substack{s_i(t)\\ i=1,\ldots,K}} \quad H(u,x). \tag{1}$$

Without loss of generality, let $s_i^*(t)$ denote the optimal critical level of class i and let $H^*(u,x)$ denote the optimal total expected cost. We know that $s_1^*(t)$ must be zero because class 1 has the highest backorder cost and there is no advantage in rejecting demand from class 1.

2.2. Dynamic critical levels for systems with two demand classes

Consider a single period system with two demand classes ($K=2$) and suppose that a customer demand has just arrived when the time remaining is t. If the demand is from class 1, it must be satisfied. If the demand is from class 2, it can either be satisfied or rejected. When this demand is satisfied, the total expected cost at the remaining time t is $H^*(t,X(t)-1)$. If this demand is rejected, the total expected cost at the remaining time t is $H^*(t,X(t))+e_2(t)$, where $e_2(t)=c_0+\pi_2+\hat{\pi}_2 t$ is the backorder cost. If $H^*(t,X(t)-1) > H^*(t,X(t))+e_2(t)$, then this demand should be rejected. Hence, the optimal dynamic critical level of class 2 is

$$s_2^*(t)=\max\{X(t) \,|\, H^*(t,X(t))+e_2(t)-H^*(t,X(t)-1)<0\}. \tag{2}$$

It is difficult to solve Eq. (2) because the closed form for $H^*(t,X(t))-H^*(t,X(t)-1)$ cannot be found easily.

Thus, we adopt two important ideas to approximate $s_2^*(t)$. First, if a demand class is rejected, this demand class will be rejected for the remaining time until the end of the period. This is a reasonable approximation because when a demand class is rejected, it