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# Optimal strategy for an integrated inventory system involving variable production and defective items under retailer partial trade credit policy

Hardik N. Soni [a,*], Kamlesh A. Patel [b]

[a] Chimanbhai Patel Post Graduate Institute of Computer Applications, Gujarat, Ahmedabad, 380015, India
[b] Mathematics Department, Shri U. P. Arts & Smt. M. G. Panchal Science & Shri V. L. Shah Commerce College, Gujarat, Pilvai, India

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ABSTRACT

This paper investigates an integrated inventory model with variable production rate and price-sensitive demand rate under two-level trade credit. The model considers two-level trade credit policy in which the retailer receives a full trade credit from its supplier, and offers partial trade credit to its customers. It is assumed that an arrival order lot may contain some defective items and the number of defective items is a random variable. This study attempts to offer a best policy for retail price, the replenishment cycle, and the number of shipment from the supplier to the retailer in one production run that aims at maximizing the joint expected total profit per unit time. An algorithm is designed to identify the optimum solution of the proposed model. Numerical examples are included to illustrate the algorithmic procedure and the effect of key parameters is studied to analyze the behavior of the model.



## 1. Introduction

Due to globalization of market and increased competition, organizations adopt trade credit policy to boost sales, promote market share, and reduce on-hand stock levels. As a result, trade credit financing plays an important role, as it can serve the source of business financing after bank or other financial institute, in business transactions. In commercial practice, the supplier usually offers a credit period to the retailer such that it allows the retailer to raise the flexibility in capital allocation. Accordingly, inventory models under trade credit have been studied extensively. Goyal [11] first derived an economic order quantity (EOQ) model under the conditions of permissible delay in payments. Aggarwal and Jaggi [2] then extended Goyal's model for deteriorating items. Jamal et al. [23] and Chang and Dye [5] extended Goyal [11] to the case for deterioration and allowable shortages. Teng [33] assumed that selling price is not equal to the purchasing price to modify Goyal [11]. Chung and Huang [10] extended Goyal [11] to consider the case that the units are replenished at a finite rate under permissible delay in payments. Since it is beyond the scope of this paper to discuss all contributions in detail we suggest to refer Soni et al. [30] for a comprehensive and up-to-date review on inventory models under trade credit.

The common characteristic in the above mentioned articles is that the supplier offers a delayed payment period to the retailer, but the retailer fails to offer the delayed payment to its customers, which is quite unrealistic. Huang [16] and Biskup et al. [4] have established an inventory model assuming that the retailer also offers a credit period to the customer which is shorter than the credit period offered by the supplier, in order to stimulate his demand. Huang [17,18] further extended Huang [16] by considering the limited storage space and finite replenishment rate, respectively. Later, Teng and Goyal [36] proposed a generalized formulation of Huang's models [16,17] and Teng and Chang [35] modified Huang [18] by relaxing the assumption that the trade credit offered by supplier is longer than trade credit offered by retailer. Liao [27] developed an economic production quantity (EPQ) model with non-instantaneous receipt and exponentially deteriorating items under the two-level trade credit policy. Chang et al. [6] modified Liao [27] by relaxing the assumption that the trade credit offered by supplier is longer than trade credit offered by retailer. In reality, the marginal effect of credit period on sales is proportional to the unrealized potential of the market demand. Incorporating this phenomenon, Jaggi et al. [21] formulated an EOQ model under two-level trade credit policy with credit-linked demand. Thangam and Uthayakumar [38] extended Jaggi et al. [21] for perishable items when demand depends on both selling price and credit period under two-level trade credit policy. Recently, Kreng and Tan [25] developed a production model for a lot-size inventory system with finite production rate and defective items which involve imperfect quality and scrap items under the condition of two-level trade credit policy. In practice, to reduce non-payment risks, a retailer frequently offers a partial trade credit to its credit risk customer who must pay a portion of the purchase amount at the time of placing an order and then avail a permissible delay on the rest of the outstanding amount. Huang and Hsu [19] developed an EOQ model in which retailer gets full trade credit but offers partial trade to the customer. Teng [34] explored optimal ordering policies for a retailer who offers distinct (i.e. full or partial trade credit) trade credits to its good and bad customers. Notice that the above studies based on two-

* Corresponding author.
*E-mail address:* hardik_soni30@yahoo.com (H.N. Soni).

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level trade credit focused only on the retailer's optimal solution, but not on both parties.

The trade credit concept on an integrated inventory model was initiated by Abad and Jaggi [1]. They considered supplier–retailer integrated system in which the supplier offers trade credit to the retailer; and determines the optimal strategies under non-cooperative and cooperative relationships for both parties. Subsequently, numerous researchers such as Chen and Kang [7,8], Ho et al. [15], Ouyang et al. [28], Teng et al. [37], Yang and Wee [39] have extended integrated inventory model in several different directions. Several interesting articles addressing the issue of supply chain management under variety of conditions are Guardiola et al. [12], Hung et al. [20], Jalbar et al. [22], Kristianto et al. [26], Sana [29], Szmerekovsky et al. [32], Yoon et al. [40], Zhang et al. [41], and others. From literature survey, there are few literatures considering integrated inventory system under two-level trade credit. For instance, Su et al. [31] presented a stylized model to determine the optimal strategy for the integrated supplier–retailer inventory model with credit-linked demand rate under the condition that both the supplier and retailer have adopted a trade credit strategy. They assumed that the retailer obtained a longer trade credit period from the supplier and provides a shorter trade credit period to customers. Generally, high selling price makes a negative impact on a major part of the customers to buy the products. That is, the market demand is inversely related to selling price. Thus, change in price affects the demand, which in turn affects the decisions on production, shipping and inventory policies. In this context, Chen and Kang [9] investigated three models considering the two-level trade credit policy with price-sensitive demand, namely a non-integrated model, integrated model and integrated vendor-buyer model with a negotiation scheme. Recently, Ho [14] extended Su et al. [31] by relaxing the assumption that the trade credit offered by supplier is longer than trade credit offered by retailer. In [14], demand rate is linked both to the retail price and the credit period offered by the retailer to the customers. In aforesaid integrated inventory models under the two-level trade credit, there are some implicit assumptions like that the fix production rate, and all the items replenished by the retailer are of perfect quality, and so on. However, those assumptions may not be fit for the real environments, and the collaborative problem with two-level trade credit needs to be addressed in a more comprehensive sense.

In this study, we develop a more general integrated supplier–retailer inventory model with a demand rate that is sensitive to the customer's price. We assume that the supplier adopts a full trade credit strategy whereas the retailer adopts a partial trade credit strategy. Besides, the production cost is assumed to be a convex function of the production rate. Additionally, it is assumed that an arrival order lot may contain some defective items and the number of defective items is a random variable. Such considerations make this paper more advantageous as compared to the existing literature. The goal of this research is to determine optimal retail price, the replenishment order, and number of shipment from the supplier to the retailer in one production run in order to maximize the joint expected total profit per unit time. An algorithm is developed to determine the optimal solution. The model is illustrated with numerical examples and sensitivity analysis with respect to key parameters is carried out.

## 2. Notation and assumptions

To develop the mathematical model, the following assumptions and notations are used.

### 2.1. Assumptions

1. There is a single-supplier and single-retailer and they deal with a single product.
2. The market demand for the item is assumed to be sensitive to the customer's retail price $s$ and is defined as $D(s)=as^{-b}$, where $a$ $(>0)$ is a scaling factor and $b$ $(>1)$ is a price elasticity. For notational simplicity, $D$ $(s)$ and $D$ will be used interchangeably in this paper.
3. The capacity utilization, $\rho$, is the ratio of the demand rate, $D$, to the production rate, $K$; it is always less than 1. (i.e. $\rho=D/K$ and $\rho<1$).
4. The same assumption used in Khouja [24] is used here. The unit production cost $c$ $(K)$ is the convex function of $K$. That is, $c\,(K)=c_0+c_1/\mathrm{K}+c_2\,\mathrm{K}$, where $c_0$, $c_1$, and $c_2$ are non-negative real numbers to be set to best fit the estimated unit production cost function. The fixed cost $c_0$ can be regarded as the raw material cost whereas $c_1$ is the direct cost like labor and energy costs and $c_2$ is tool wear and tear cost. For notational simplicity, $c$ $(K)$ and $c$ will be used interchangeably in this paper (cf. [28]).
5. An arrival may contain some defective items. The number of defective items in an arriving order of size $Q$ is a binomial random variable with parameters $Q$ and $p$. Upon arrival of an order, all the items are inspected and defective items in each lot will be returned to the vendor at the time of delivery of the next lot (cf. [3]).
6. Inspection is non-destructive and error-free. The inspection process is considered promptly and gives the retailer the ability to scan an entire lot efficiently and effectively. From this perspective, length of inspection period is neglected here (cf. [13])
7. The two-level trade credit policy is adopted. The supplier provides a full trade credit period $M$ to the retailer to settle the entire purchase cost while the retailer offers a partial trade credit period $N$ to its credit-risk customers.
8. Fraction $(\alpha)$ of the purchase cost in which the customer must pay the retailer at the time of placing an order, with $0\leq\alpha\leq1$. Hence, $(1-\alpha)$ is the portion of the purchase cost in which the retailer offers its customer a permissible delay of $N$ periods.
9. Before the settlement of an account, the retailer can use sales revenue to earn the interest with an annual rate $Ie$ up to the end of period $M$. At time $t=M$, the credit is settled and the retailer starts to pay the interest at rate $Ic$ for the items in stock. During the period of delayed payment, the supplier has an opportunity interest loss with the annual rate $Iv$.
10. Shortages are not allowed and lead time is negligible.
11. The system operates for an infinite planning horizon.

Some additional notations are as follows:

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|---|---|
| $m$ | Number of shipment from supplier to retailer. |
| $s$ | Per unit maximum retail price (MRP) of non-defective items. |

For Supplier:

| | |
|---|---|
| $A_s$ | Set-up cost. |
| $w_s$ | Unit treatment cost (include warranty cost) of defective goods. |
| $r_1$ | Holding cost rate excluding interest charge per unit per unit time. |

For Retailer:

| | |
|---|---|
| $A_r$ | Ordering cost. |
| $v$ | Procurement cost per unit item. $(s>v>c)$ |
| $r_2$ | Holding cost rate excluding interest charge per unit per unit time for non-defective item. |
| $r_3$ | Holding cost rate excluding interest charge per unit per unit time for defective item $(r_3<r_2)$. |
| $p$ | Defective rate in an order lot, $p\in[0,1)$, a random variable. |
| $f(p)$ | The probability density function (p.d.f.) of $p$ with finite mean $E$ $(p)$ and variance $V_p$, where $E(p)=\int_0^1 pf(p)dp$. |
| $c_{sc}$ | The screening cost per unit. |
| $c_t$ | Fix transportation cost per shipment. |
| $T$ | Replenishment cycle time. |
| $Q$ | Order quantity. |
| $E(\cdot)$ | Expected value operator. |