Int. J. Production Economics 143 (2013) 35–40

Contents lists available at ScienceDirect

Int. J. Production Economics

journal homepage: www.elsevier.com/locate/ijpe

Optimal MRP parameters for a single item inventory with random replenishment lead time, POQ policy and service level constraint

Mohamed-Aly Louly [a], Alexandre Dolgui [b,*]

[a] FARCAMT, King Saud University, College of Engineering, Industrial Engineering Department, P.O. Box 800, Riyadh 11421, Saudi Arabia
[b] Ecole des Mines de Saint-Etienne, Centre for Industrial Engineering and Computer Science, Laboratory for Information Science and Technology, 158 cours Fauriel, 42023 Saint-Etienne Cedex 2, France

ARTICLE INFO

Article history:
Received 15 December 2009
Accepted 7 February 2011
Available online 17 February 2011

Keywords:
Supply planning
Stochastic lead time
Periodic Order Quantity
MRP parameterization

ABSTRACT

This study deals with Material Requirement Planning (MRP) software parameterization under uncertainties. The actual lead time has random deviations; so it can be considered as a random variable. MRP approach with Periodic Order Quantity (POQ) policy is considered. The aim is to find the optimal MRP time phasing corresponding to each periodicity of the POQ policy. This is a crucial issue in supply planning with MRP approach because inappropriate planned lead times under lead time uncertainties invariably lead to large and costly inventories or insufficient customer service levels. The proposed model and algorithms minimize the sum of the setup and holding costs while satisfying a constraint on the service level. Our approach does not need to employ the commonly used normal probability distributions. Instead, its originality is in finding a closed form of the objective function, valid for any probability distribution of the actual lead times.

1. Introduction

Effective replenishment is a crucial problem in supply planning. An inadequate inventory control policy leads to overstocking or stockout situations. In the former, the generated inventories are expensive and in the latter there are shortages and penalties due to unsatisfied customer demands.

Material Requirements Planning (MRP) is a commonly accepted approach for replenishment planning in major companies (Axsäter, 2006). The MRP-based software tools are accepted readily. Most industrial decision makers are familiar with their use. The practical aspect of MRP lies in the fact that this is based on comprehensible rules, and provides cognitive support, as well as a powerful information system for decision making. Some instructive presentations of this approach can be found in Baker (1993), Sipper and Bulfin (1998), Zipkin (2000), Axsäter (2006), Tempelmeier (2006), Dolgui and Proth (2010) and Graves (2011).

Nevertheless, MRP is based on the supposition that both demand and lead time are deterministic. However, most production systems are stochastic. For example, a random lead time can be explained by the variability of actual supplier load (when a supplier furnishes several clients, its load depends on the timing of all client orders, if total demand outstrips production capacity, the lead time increases). There are many other external factors increasing randomness of lead times: outsourced production overseas can introduce some randomness via shipping perturbations, the orders might not arrive by the due date because of work stoppage or delays attributable to the weather (Graves, 2011). Additional random factors and unpredictable events such as machine breakdowns, absenteeism, other random variations of capacity can cause deviations in actual lead times from planning ones (Koh and Saad, 2003; Chaharsooghi and Heydari, 2010). Therefore, as aforementioned the deterministic assumptions of MRP can be often too restrictive.

As shown in Whybark and Wiliams (1976), Ho and Lau (1994), Molinder (1997) and Chaharsooghi and Heydari (2010), lead time is a principal factor foreseeing production and lead time randomness affects seriously ordering policies, inventory levels and customer service levels.

Thankfully, the MRP approach can be tailored to uncertainties by searching optimal values for its parameters (Buzacott and Shanthikumar, 1994; Hegedus and Hopp, 2001; Koh and Saad, 2003; Inderfurth, 2009; Mula and Poler, 2010). An adequate choice of these parameters increases the effectiveness of MRP techniques. Thus, one of the essential issues for companies in industrial situations is MRP parameterization. This is commonly called MRP offsetting under uncertainties.

There are several MRP parameters: planned lead time, safety stock, lot-sizing rule, freezing horizon, planning horizon, etc. There exist extensive publications concerning safety stock

* Corresponding author. Tel.: +33 4 77 42 01 66; fax: +33 4 77 42 66 66.
E-mail addresses: louly@ksu.edu.sa (M.-A. Louly), dolgui@emse.fr (A. Dolgui).
URL: http://www.emse.fr/~dolgui (A. Dolgui).

0925-5273/$ - see front matter
doi:10.1016/j.ijpe.2011.02.009

calculation for random demand of finished products (Porteus, 1990; Lee and Nahmias, 1993). In contrast, certain parameters seem not to be sufficiently examined as, for example, planned lead time (differences between due dates and release date). Optimal parameterization of most used lot-sizing rules is also an open issue.

If actual lead time is random, the planned lead time can contain safety lead time, i.e. the planned lead time is calculated as the sum of the forecasted (or contracted) and safety lead times. The latter should be formulated as a trade-off between overstocking and stockout while minimizing the total cost. The search for optimal value of safety lead time, and, consequently, for planned lead time, is a crucial issue in supply planning with the MRP approach. The problem of planned lead times optimization, when safety lead times are used, has been given scant attention in the literature. In practice, often average values or percentiles of probability distributions of actual lead times are used. Nevertheless, a longer than necessary planned lead time creates excessive work in progress. Perhaps of special interest, Graves (2011) in his chapter 'Uncertainties and Production Planning' of the 'Handbook of Production Planning and Inventories in the Extended Enterprise' considers that there is "*a great opportunity for developing decision support to help planners in understanding the trade-offs and in setting these parameters in a more scientific way*". This is one of motivations for this paper where we propose a decision support model for optimal MRP time phasing for each periodicity of the POQ policy. The proposed model and algorithms minimize the sum of the setup and holding costs while satisfying a constraint on the service level. Our approach does not need to employ the commonly used normal probability distributions. Instead, its originality is in finding a closed form of the objective function, valid for any probability distribution of the actual lead times.

2. Basic principles of MRP systems

The goal of Material Requirements Planning (MRP) is to determine a replenishment schedule for a given time horizon. The MRP approach deals with the calculation of these requirements for a series of sequential planning periods (time buckets). One time bucket can be equal to a day, week or month depending on the applications. The gross requirements for the finished product are given by the Master Production Schedule (MPS). The net requirements and planned order releases are deduced from gross requirements and projected on-hand inventory.

Let us introduce the following notations:

x planned lead time,
$Q(i-x)$ planned orders released at time bucket $i-x$, and for time bucket i:
$I(i)$ projected on-hand inventory,
$N(i)$ net requirements,
$G(i)$ gross requirements.

The initial inventory $I(0)$ is assumed to be known.

The net requirements for time bucket i are obtained as follows:

$$N(i) = G(i) - I(i-1)$$

The planned released order quantity:

$$Q(i-x) = \max\{0, N(i)\}$$

This is the core of the MRP approach. These rules are implemented in MRP tables.

The above formulas concern the Lot-for-Lot (L4L) policy, i.e. where the orders are not grouped. Often the simple L4L policy is not possible (transportation constraints,…) or too costly (because of an expensive setup,…), so another lot-sizing rule should be used. Here, we consider the Periodic Order Quantity (POQ) policy that consists of grouping orders for p consecutive periods. In this case, the planned release order quantity is calculated as follows:

$$Q(i-x) = \max\left\{0, \sum_{j=0}^{p-1} G(i+j) - I(i-1)\right\}$$

Periods (time buckets)	1	2	3	4	5	6	7	8
Gross requirements	40	35	65	45	20	55	45	35
Projected On-hand 100	60	25	45	0	55	0	35	0
Planned Order receipts			85		75		80	
Planned Order Releases		85		75		80		

Fig. 1. An example of MRP table for POQ policy.

An example of MRP table for the POQ policy is given in Fig. 1.

Each MRP table has several parameters: periodicity for grouping orders (for POQ policy), planned lead time for time phasing, safety stock (if necessary), etc. In this paper, only the following two essential parameters are considered:

– periodicity (p) and
– planned lead time (x)

In Fig. 1, these parameters are equal to $p=2$ and $x=1$. In a stochastic environment, an adequate choice of these parameters is crucial, because this defines the average total cost of supply planning. In general, the calculation of the optimal values of these parameters is a very complex optimization problem.

Note that in literature, another parameter is often studied, especially for a random demand, which is safety stock. This parameter is not considered here (safety stock is set to 0). Some discussions on safety lead time versus safety stock (the former is included in calculation of the planned lead time x) are reported in the next section.

3. Related publications

We have chosen to limit this section to some related publications and key articles illustrating the problem, its history and major results. For more exhaustive reviews on the simulation and analytical studies of MRP parameters under uncertainties, see Yeung et al. (1998), Koh and Saad (2003), Mula et al. (2006), Dolgui and Prodhon (2007), Inderfurth (2009) and Louly and Dolgui (2010).

In general, the majority of publications are devoted to the MRP parameterization under customer demand uncertainties. Simulations are more often used. As to random lead times, the number of publications is relatively small.

In this paper, the attention will be focused only on lead time uncertainties. For this case, some earlier publications have already considered a calculation of safety lead time via simulation. For example, the simulation experiments by Whybark and Wiliams (1976) show that under lead time uncertainty one is better offsetting safety times than safety stocks. Ho and Lau (1994) compared different lot-sizing rules. Molinder (1997) proposes a simulated annealing approach to find appropriate safety lead time in a simulation/optimization approach. Enns (2001) showed that by selecting proper lot-sizes and planned lead times the inventory levels can be diminished. An alternative of MRP for the case of both demand and lead-time uncertainties was considered by Mohebbi et al. (2007). Using also simulation, the authors have shown that their approach, called capacity