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Finite horizon stochastic inventory problem with dual sourcing: Near myopic and heuristics bounds

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ABSTRACT

We consider a class of multi-periodic non-stationary stochastic single-product inventory planning problems where two procurement modes can be used at each period: a first order with immediate delivery and a second order with a single-period delivery delay. Clearly, the slow delivery mode is less expensive than the fast. We develop a discounted backlog model, with non-stationary procurement, inventory holding and backlog penalty costs proportional to the ordered quantities, inventory levels and number of backlogged units respectively. The demands are defined as non-stationary and independent random variables. We partially characterize the optimal ordering policy structure and we develop theoretical bounds and heuristic approximations for this optimal policy. Efficiency of these approximations is illustrated via extensive numerical experiments.

1. Introduction

Nowadays, in supply chains, the gap between procurement costs from different suppliers may be important, especially in the case where suppliers production sites are located in different countries. This is particularly due to the difference in raw materials and workforce costs between the production sites. Often, the difference in procurement costs goes hand in hand with the difference in delivery lead times: the longer the delivery lead time, the lower the unit ordering cost.

In this paper, we model this issue in a stochastic multi-periodic inventory planning framework with backlog where a retailer can order twice at each period of the horizon: the first order is made using a fast procurement mode with immediate delivery, while the second order is made using a slow, but cheaper procurement mode that has a single-period delivery lead time. Unsatisfied orders at a given period are backlogged to be fulfilled in the next period. We associate this dual procurement process with a discounted cost inventory model with procurement, inventory holding and backlog penalty costs, which are non-stationary and proportional to the ordered quantities, inventory levels and number of backlogged units respectively. The random demands are assumed to be independent with possibly non-stationary probability distributions.

From a global perspective, two main research trends are related to this class of problems. A first trend of papers analyzes the theoretical properties holding for such multi-ordering inventory systems (as the convexity of the cost function or the theoretical structure of the optimal policies) and studies, mainly under time-stationarity assumption, how classical inventory policies for single ordering systems (periodical review or continuous review policy) can be adapted to such multi-ordering systems. A second stream of research considers heuristics for non-stationary multi-periodic single ordering problems. This paper can be viewed in some sense at the crossing of these research streams. Indeed, first we extend and adapt some known theoretical results holding for non-stationary single ordering mode inventory models to the class of slow/fast ordering modes setting. Then using the methodology provided in (Morton and Pentico, 1995; Anupindi et al., 1996) for a single procurement mode inventory model with lost sales, we generalize their approach to our two delivery modes backlog inventory setting. We show how the optimal ordering policy can be approximated by a classical non-stationary order-up-to inventory policy and we give numerical computation procedure in order to estimate heuristic approximations for the order-up-to levels, based on upper and lower bounds. A numerical analysis is provided in order to compare the approximate policies obtained from these bounds to the optimal solution numerically computed by a stochastic dynamic programming approach.

The rest of this paper is structured as follows. In Section 2, we provide a literature review for the considered inventory

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setting. In Section 3, we introduce the model and the parameters. In Section 4, we first present an equivalent formulation via a classical cost transformation and we provide theoretical optimality conditions. The theoretical upper and lower bounds and associated heuristic approximations are presented in Sections 5 and 6. In Section 7, we provide a numerical study showing efficiency of the proposed approximations. Finally, we conclude and give some new research perspectives.

2. Literature review

Several papers in the operations management literature look at inventory models with multiple supply modes with different lead times and costs. As a pioneering research, Fukuda (1964) analyzed an inventory system where orders are placed to be delivered after a time lag of k and $k+1$ review periods, with a higher cost for faster delivery. Neuts (1964) studied periodic review inventory management models with two replenishment modes: emergency mode with immediate delivery and regular mode with a single-period delivery lead time. Whittemore and Saunders (1977) focused on theoretical conditions under which the optimal solution consists of using a single supply mode (fast or slow). Chiang and Gutierrez (1996) and Tagaras and Vlachos (2001) extended the classical periodic review model to a model in which an emergency supply mode can be used in addition to the regular supply channel. Janssen and de Kok (1999) studied an inventory system with two suppliers where a supply agreement is drawn up with one of the suppliers in order to deliver a fixed quantity Q every review period. The replenishment decisions for the other supplier are governed by a (R, S) replenishment policy. In addition to several ordering modes in each period, Sethi et al. (2003) generalized the setting by introducing demand forecast updates. Xu (2008) proposed a two-product multi-periodic supply contract model in which a buyer orders from a supplier a quantity that can be delivered with two different supply modes: the expedited one delivers the products immediately and the regular one requires a one-period delivery lead time. Allon and Van Mieghem (2010) discussed a firm that can use two sourcing options: a responsive nearshore source and a low-cost offshore source. The firm must determine an inventory sourcing policy to satisfy random demand over time. They analyzed a tailored base-surge sourcing policy combining push and pull controls by replenishing at a constant rate from the offshore source and producing at the nearshore plant only when inventory is below a target. Boute and Van Mieghem (2011) analyzed a global dual sourcing policy, where a company has access to two suppliers with complementary competencies: a local supplier that is responsive but more expensive, and a global supplier that is most cost-efficient but with a longer lead time. In their paper, the authors provided a global dual sourcing policy that allows to design an ordering policy that allocates the order volume to both sources so as to optimally trade-off cost and responsiveness. Veeraraghavan and Scheller-Wolf (2008) examined a possibly capacitated, periodically reviewed, single-stage inventory system where replenishment can be obtained either through a regular fixed lead time channel, or, for a premium, via a channel with a smaller fixed lead time. They consideredthe case when the unsatisfied demands are backordered over an infinite horizon. Sheopuri et al. (2010) extended the works of Veeraraghavan and Scheller-Wolf (2008) and studied an inventory system under periodic review in the presence of two suppliers with an emergency supplier having a shorter lead-time than a regular supplier but a higher price. They proposed two classes of policies: the first class consists of policies that have an order-up-to structure for the emergency supplier and the second class consists of policies that have an order-up-to structure for the regular supplier. Zhang et al. (2012) looked at a dual-sourcing inventory system, where procuring from one supplier involves a high variable cost but negligible fixed cost whereas procuring from the other supplier involves a low variable cost but high fixed cost, as well as an order size constraint. Scheller-Wolf et al. (2009) addressed a periodic-review inventory system with backorders and dual sourcing in which replenishment can occur through a regular channel and/or a more expensive expedited channel with a smaller lead time. To solve this problem, they introduced a class of heuristic base stock policies called the single index policies, since they require maintaining only one system statistic. Arts et al. (2011) examined a similar problem with two supply sources facing stochastic demand. In their model, a premium is paid for each product ordered from the faster emergency supply source. They studied a type of base-stock policy known as the dual-index policy as control mechanism for their inventory system. Jain et al. (2011) explicitly analyzed the impact of fixed costs on the optimal policy structure in a two ordering mode setting with information update. For a detailed survey of this literature, we refer readers to Jain et al. (2011), Minner (2003) and Thomas and Tyworth (2006).

Another category of the literature focused on dual or multiple sourcing inventory systems with stochastic lead times (Ganeshana et al., 1999; Kelle and Miller, 2001; Ryu and Lee, 2003; Arda and Hennet, 2006).

However, theoretical properties and approximate solution methods for stochastic non-stationary multi-periodic inventory models have been analyzed in numerous papers. As a pioneering work, Veinott (1965) established, under suited assumptions, some link between the optimal policy and myopic policies. Morton (1978) extended these results by exhibiting a sequence of bounds for the optimal policy and by giving planning horizon results. This author proposed improved near-myopic bounds and heuristic near-optimal inventory policies in Morton and Pentico (1995) and Anupindi et al. (1996). For a detailed survey of this literature, we refer readers to Iida (2001) and Levi et al. (2008).

3. The model

3.1. The procurement/inventory system

Consider a N time-periods non-stationary procurement/inventory problem. For each period, a single-product stochastic demand is defined by a probability density function with known parameters. The random demands are assumed to be independent. A pair of orders can be issued at the beginning of each period: a first quantity immediately delivered through a fast procurement mode and a second quantity, delivered via a slow mode, at the beginning of the next period. The slow procurement mode is assumed to be less expensive than the fast one. At the end of each period, unsatisfied demands are backlogged to be delivered in a future period, and a backlog penalty cost, proportional to the number of backlogged units, is charged. The remaining inventory, if any, is kept for the next period at a holding cost proportional to the inventory level available at the end of the period. At the end of the last period, the remaining inventory is salvaged, while backlogs are delivered by a last fast order.

Model notations are the following. The random demand at period t is D_t, for $t=1,\dots,N$, with $F_t(\cdot)$, the cumulative distribution function of D_t and $F_{t,N}(\cdot)$ the cumulative distribution of demand over periods t through N. Define $D_{t,j}=\sum_{k=t}^{j} D_k$, with $D_{t,t-1}=0$.

The decision variables are

- Q_t: the quantity ordered and delivered at the beginning of period t, for $t=1,\dots,N+1$,