



# A note on periodic review inventory models with stochastic supplier's visit intervals and fixed ordering cost



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## ABSTRACT

Most periodic review models in the inventory literature have assumed a fixed length of the review periods. In this note, we extend the work of Chiang (2008), and consider backlogged and lost-sales periodic review models where the review periods are of a variable length and there is a fixed cost of ordering for replenishment. Assuming that period lengths are independently and identically distributed, we show (using an exact method of computing inventory holding costs) that an  $(s, S)$  policy is optimal for the infinite horizon problem. The periodic review policies developed are thus easy to implement. The computation shows that if the fixed cost of ordering is small, one needs to use the proposed periodic policies.

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## 1. Introduction

Most periodic review systems in the inventory-control literature have assumed a fixed length of the review periods. It is possible in practice that periodic systems have the review periods of a variable length. Such systems arise mainly from supply uncertainties. For example, Chiang (2008) observed that many supermarkets have suppliers who come to visit regularly and replenish inventories for them. However, the supplier does not always come in constant time intervals. Depending on her visit plans or work schedules, she often arrives at a particular supermarket earlier or later than planned. The elapsed time between two consecutive visits varies in nature. See also Ertogral and Rahim (2005) for supply chain settings where the replenishment epochs are not under the retailer's control (i.e., under the supplier's control), and Tang and Musa (2011) for a variety of supply chain risks or uncertainties.

To the best of our knowledge, the issue of the period length variability or replenishment interval randomness is investigated only recently by Ertogral and Rahim (2005) and Chiang (2008). Ertogral and Rahim (2005) derived the expected profit per replenishment cycle by assuming constant demand; Chiang (2008) used dynamic programming to develop periodic review inventory models with stochastic demand. However, these studies assumed that the fixed cost of ordering for replenishment is zero.

In this paper, we extend the work of Chiang (2008) and incorporate a fixed cost of ordering. It is possible that the supplier visits a retailer and charges a service expense if the retailer's

inventory is replenished. Moreover, instead of using an approximate method as in Ertogral and Rahim (2005) and Chiang (2008), we use an exact approach of computing inventory holding costs. We assume that period lengths are independently and identically distributed (iid), as in the above two studies, and examine both the backlogged and lost-sales periodic review inventory problems. We will show that the optimal policy is of the  $(s, S)$  type. Hence, existing algorithms (e.g., Zheng and Federgruen, 1991) could be used to find the optimal  $s$  and  $S$ . The periodic review models developed can be viewed as a generalization of ordinary periodic models where the period length is fixed.

The computation shows that when the fixed cost of ordering is small (but not small enough to be neglected, such that an order is always placed at a review epoch), ignoring the period length variability can incur unnecessary large losses, especially if lead-time is zero, shortage is costly, demand variability is small, and/or the period length is volatile. These results agree with Chiang (2008). Hence, one needs to use the proposed ordering policies, and the suggestions made in Chiang (2008) apply here, e.g., if the replenishment epochs are under the supplier's control, the retailer should somehow persuade the supplier to visit more regularly, or even cooperate or form a strategic alliance with the supplier in the long run; Prajogo et al. (2012) recently showed that strategic long-term relationship, one of the three supplier management practices suggested, has a positive relationship with a firm's operational performance, and Cheng et al. (2012) found that a purchasing firm tends to form *quanxi* networks with its key supplier to improve communication and thus reduce supply risk. However, when the fixed ordering cost is large, ignoring the period length variability causes small or virtually no losses to a firm, especially if lead-time is long or demand variability is large. The implication of this is that

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it is alright to use the ordinary periodic review models in the case of large fixed ordering costs.

**2. Backlogged periodic review inventory models**

We first consider the case where demand not satisfied at once is backlogged. We use the same notation as in Chiang (2008). Demand is stochastic with mean rate  $\mu$  per day, and is assumed to be non-negative and independently distributed in disjoint time intervals. Let  $T$  denote the period length and  $D$  the demand during  $T$ . Successive  $T$ 's are assumed to be iid random variables. Let  $\phi(\cdot)$  be the probability density function (pdf) of  $T$  and  $g(\cdot|T)$  be the conditional pdf of  $D$ . Also, let  $c$  be the unit purchase cost,  $\alpha$  the discount rate,  $y$  the inventory position (i.e., inventory on hand minus backorder plus inventory on order) after a possible order is placed at a review epoch, and  $H$  the expected one-period inventory holding and shortage costs ( $H$  is a function of  $y$ ).

We assume without loss of generality (also for simplicity) that replenishment is immediate. The case of a positive (constant) lead-time  $L$  can be handled by appropriately redefining  $H$ , i.e., given time 0 at a review epoch,  $H$  is charged for the time interval  $[L, T + L]$  (see Chiang, 2008 or Porteus, 1990). Let  $K$  denote the fixed cost of ordering and  $V_n(x)$  the expected discounted cost with  $n$  periods remaining until the end of the planning horizon when the starting inventory position is  $x$  and an optimal ordering policy is used at every review epoch.  $V_n(x)$  satisfies the functional equation

$$V_n(x) = \min_{x \leq y} \{K\delta(y-x) + cy + H(y) + E_T[e^{-\alpha T} E_{D|T}[V_{n-1}(y-D)]]\} - cx, \tag{1}$$

where  $\delta(\cdot)$  is the Dirac-delta function that is equal to 1 if the argument is positive and 0 otherwise,  $c(y - x)$  is the procurement cost, and  $E_T[e^{-\alpha T} E_{D|T}[V_{n-1}(y - D)]]$  is the expected discounted cost from the next review epoch to the end of the planning horizon. Since  $T$  is stochastic, the planning time horizon in (1) is of a random length, as opposed to the fixed time horizon of  $N$  periods commonly studied in the literature. Eq. (1) is basically the same as expression (1) of Chiang (2008), except that a fixed cost  $K$  is presented.

Let  $\beta = E_T[e^{-\alpha T}]$ . Define  $\varphi(\cdot) \equiv E_T[e^{-\alpha T} g(\cdot|T)]/\beta$ , i.e.,  $\beta\varphi(\cdot) = E_T[e^{-\alpha T} g(\cdot|T)]$  is the discount density of  $D$  and  $\varphi(\cdot)$  is the “normalized” pdf of  $D$ . Chiang (2008) showed that

$$\begin{aligned} E_T \left[ e^{-\alpha T} E_{D|T} \left[ V_{n-1}(y-D) \right] \right] &= \int_0^\infty e^{-\alpha T} \left( \int_0^\infty V_{n-1}(y-D) g(D|T) dD \right) \phi(T) dT \\ &= \beta \int_0^\infty V_{n-1}(y-D) \left\{ \left( \int_0^\infty e^{-\alpha T} g(D|T) \phi(T) dT \right) / \beta \right\} dD \\ &= \beta E_D \left[ V_{n-1}(y-D) \right] \end{aligned} \tag{2}$$

where the expectation  $E_D$  is taken over the pdf  $\varphi(\cdot)$ . Thus,  $V_n(x)$  can be written by

$$V_n(x) = \min_{x \leq y} \{K\delta(y-x) + cy + H(y) + \beta E_D[V_{n-1}(y-D)]\} - cx. \tag{3}$$

There are a few approaches in the inventory literature of computing the one-period cost function  $H(y)$ . Chiang (2008) used an approximate method based on Hadley and Whitin (1963, pp. 237–239). In this paper, we adopt an exact approach that charges the holding and shortage costs based on inventory on hand and backlogged demand, respectively at the end of each period. Let  $h'$  be the holding cost per unit per period (irrespective of its length), and  $p$  the shortage cost per unit per period. If  $T$  is constant,  $H(y)$  is expressed by

$$H(y) = \int_0^{y+} h'(y-D)g(D|T) dD + \int_{y+}^\infty p(D-y)g(D|T) dD, \tag{4}$$

where  $(\cdot)^+ \equiv \max\{\cdot, 0\}$ . For the present model in which  $T$  is stochastic,  $H(y)$  is given by

$$H(y) = \int_0^{y+} h'(y-D)g^*(D) dD + \int_{y+}^\infty p(D-y)g^*(D) dD \tag{5}$$

where  $g^*(\cdot) \equiv E_T[g(\cdot|T)]$ . However, since the length of a period is not constant, the holding cost could be computed in proportion to it. Let  $h$  be the holding cost per day per unit held at the end of a period. Then  $H(y)$  is given by

$$H(y) = \int_0^{y+} hE[T](y-D)g'(D)dD + \int_{y+}^\infty p(D-y)g^*(D)dD, \tag{6}$$

where  $g'(\cdot) \equiv E_T[Tg(\cdot|T)]/E[T]$ . Note that the two pdf's  $g^*(\cdot)$  and  $g'(\cdot)$  do not differ much, especially if  $T$ 's variability is not large, and  $h'$  should be equal to  $hE[T]$ .

We can easily verify that  $H(y)$ , given by either (5) or (6), is convex. Thus, we have the following theorem.

**Theorem 1.** *The optimal policy for  $V_n(x)$  in (3) is of the (s, S) type.*

**Proof.**  $H(y)$  is convex and  $V_n(x)$  is in the form of expression (1) of Iglehart (1963). □

Hence, a stationary (s, S) policy is optimal for the infinite horizon model; in other words, if  $x \leq s$ , order the amount  $S - x$ ; otherwise if  $x > s$ , do not order. Define

$$G(y) \equiv cy(1-\beta) + H(y). \tag{7}$$

To compute the two optimal operational parameters  $s^*$  and  $S^*$ , we use  $G(y)$  and the discount renewal density  $\beta\varphi(\cdot)$  in a solution procedure (e.g., Veinott and Wagner, 1965). If  $\alpha = 0$  (i.e., the undiscounted-cost criterion is used), we use  $H(y)$  and the density  $E_T[g(\cdot|T)]$  instead. For discrete demand distributions, Zheng and Federgruen (1991) had developed an efficient algorithm to obtain  $s^*$  and  $S^*$ , along with the long-run average cost  $C(s, S)$ . If  $T$  is constant, denote by  $s'$  and  $S'$  the two optimal parameters obtained.

If  $K = 0$ , an order-up-to policy is optimal and the optimal  $S^*$  is found by minimizing  $G(y)$  (Veinott and Wagner, 1965), which is the expected cost of the upcoming period (not including the constant procurement cost  $c\beta E_D[D]$ ). In other words,  $S^*$  is the solution to

$$\int_S^\infty g^*(D)dD = [c(1-\beta) + h']/(h' + p) \tag{8}$$

if  $H(y)$  given by (5) is used, or the solution to

$$\int_S^\infty hE[T]g'(D)dD + \int_S^\infty pg^*(D)dD = c(1-\beta) + hE[T] \tag{9}$$

if  $H(y)$  given by (6) is used. Since  $p$  should be greater than  $c(1 - \beta)$ ,  $S^*$  is guaranteed to be obtained.

**3. Lost-sales periodic review inventory models**

Next, we consider the situation where demand not satisfied at once is lost. Assume that replenishment is immediate. Let  $z$  be the order quantity placed at a review epoch and redefine  $x$  as the starting on-hand inventory. Then  $V_n(x)$  satisfies the recursive equation

$$\begin{aligned} V_n(x) &= \min_{z \geq 0} \{K\delta(z) + cz + H(x+z) + E_T[e^{-\alpha T} E_{D|T}[V_{n-1}((x+z-D)^+)]]\} \\ &= \min_{x \leq y} \{K\delta(y-x) + cy + H(y) + E_T[e^{-\alpha T} E_{D|T}[V_{n-1}((y-D)^+)]]\} - cx. \end{aligned} \tag{10}$$

Eq. (10) is the same as expression (9) of Chiang (2008), except that a fixed ordering cost is presented. Through a transformation as in (2), we can rewrite (10) by

$$V_n(x) = \min_{x \leq y} \{K\delta(y-x) + cy + H(y) + \beta E_D[V_{n-1}((y-D)^+)]\} - cx. \tag{11}$$

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