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Due-date assignment and maintenance activity scheduling problem

Gur Mosheiov[a,*], Daniel Oron[b]

[a] *School of Business Administration and Department of Statistics, The Hebrew University, Jerusalem 91905, Israel*
[b] *Econometrics and Business Statistics, The University of Sydney, Australia*

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Abstract

In the scheduling problem addressed in this note we have to determine: (i) the job sequence, (ii) the (common) due-date, and (iii) the location of a rate modifying (maintenance) activity. Jobs scheduled before (after) the due-date are penalized according to their earliness (tardiness) value. The processing time of a job scheduled after the maintenance activity decreases by a job-dependent factor. The objective is minimum total earliness, tardiness and due-date cost. We introduce a polynomial ($O(n^4)$) solution for the problem.

Keywords: Scheduling; Single machine; Due-date assignment; Earliness–tardiness; Rate modifying activity

1. Introduction

Lee and Leon [1] introduced a new class of scheduling problems. In these problems, in addition to the traditional (job scheduling) decisions, a decision has to be made regarding a *rate modifying activity*. This activity affects the performance of the machine, i.e., it affects the production time of the following jobs. Both positive and negative effects may occur. However, quite often the rate modification is due to *maintenance*, implying that the machine will only improve, i.e., the production rate will increase. It is obvious that during the maintenance period, the machine is idle and no production takes place. Thus, when scheduling a maintenance activity, one has to consider the tradeoff between the temporary machine shutdown, and the improvement in the production rate. Lee and Leon [1] studied several single machine scheduling problems in this class: minimizing makespan, flow-time, weighted flow-time and maximum lateness. Mosheiov and Sidney [2] addressed the problems of minimizing makespan with precedence relations, minimizing makespan with learning effect, and minimizing the number of tardy jobs.

In this note we study a classical *due-date assignment* problem with the option of scheduling a maintenance activity. Panwalker, Smith and Seidmann [3] addressed the following single machine scheduling and common due-date assignment problem: "...All jobs have a common (but unknown) due-date. The objective is to find an optimal value of the due-date *and* optimal sequence which minimizes the total penalty based on the due-date value and the earliness or tardiness of each job". Panwalker et al. consider a set of n jobs available at time zero. The common due-date d is a decision variable. The processing time of job j is denoted by p_j, $j = 1, \ldots, n$. For a given schedule, the

* Corresponding author. Tel.: +972 2 588 3790; fax: +972 2 588 1341.
E-mail address: msomer@mscc.huji.ac.il (G. Mosheiov).

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completion time of job j is denoted by C_j. The earliness and tardiness of job j are defined as $E_j = \max\{d - C_j, 0\}$ and $T_j = \max\{C_j - d, 0\}$, $j = 1, \dots, n$, respectively. Three cost components are assumed: for earliness, for tardiness and for (delaying the) due-date. The unit penalties for earliness, tardiness and due-date are denoted by α, β and γ, respectively.

The objective is to minimize the total cost, i.e.,

$$Z = \sum_{j=1}^{n} (\alpha E_j + \beta T_j + \gamma d) = \alpha \sum_{j=1}^{n} E_j + \beta \sum_{j=1}^{n} T_j + n\gamma d. \tag{1}$$

Panwalker et al. [3] presented an $O(n \log n)$ solution for the problem.

When considering the option of performing a rate modifying activity, both the optimal job sequence and the complexity of the solution algorithm may change significantly. Both issues are investigated in this note. The rate modifying activity is denoted by rm, and (as in Lee and Leon [1]) the length of rm is denoted by t. The processing of job j if scheduled after rm is $\delta_j p_j$, $j = 1, \dots, n$, where $\delta_j > 0$ is the modifying rate. (Although the solution presented in this note is valid for any positive δ-value, as mentioned above, in most applications, namely when scheduling a maintenance activity, $\delta_j < 1$.) A solution of the problem consists of finding: (i) the optimal job sequence, (ii) the optimal due-date, and (iii) the optimal timing of the rate modifying activity. Using the conventional notation, the problem studied in this note is $1/rm/\sum_{j=1}^{n}(\alpha E_j + \beta T_j + \gamma d)$. We show that the problem is solvable in polynomial $(O(n^4))$ time.

2. An optimal solution for $1/rm/\sum_{j=1}^{n}(\alpha E_j + \beta T_j + \gamma d)$

In the first part of this section, we show that several properties of an optimal solution for the original due-date assignment problem, proved by Panwalker et al. [3], continue to hold when a rate modifying activity is assumed. In Property 1 we prove that an optimal schedule exists in which the due-date coincides with a job completion time.

Property 1. *An optimal schedule exists in which $d = C_j$ for some j.*

Proof. Assume that an optimal schedule exists such that $C_{k-1} \le d \le C_k$ for some job k. Assume also that rm is scheduled after the ith job. We focus in this proof on the case where $i < k$, i.e., the rate modifying is scheduled prior to the due-date. (The complementary case is proved similarly.)

Let $\Delta = d - C_{k-1}$. Note that $0 \le \Delta \le \delta_k p_k$.

The earliness cost associated with job j, $j = k-1, k-2, \dots, 1$ (denoted by Z_j), is given by

$$\begin{aligned}
&Z_{k-1} = \alpha\Delta \\
&Z_{k-2} = \alpha(\Delta + \delta_{k-1}p_{k-1}) \\
&Z_{k-3} = \alpha(\Delta + \delta_{k-1}p_{k-1} + \delta_{k-2}p_{k-2}) \\
&\vdots \\
&Z_{i+1} = \alpha(\Delta + \delta_{k-1}p_{k-1} + \delta_{k-2}p_{k-2} + \cdots + \delta_{i+2}p_{i+2}) \\
&Z_i = \alpha(\Delta + \delta_{k-1}p_{k-1} + \delta_{k-2}p_{k-2} + \cdots + \delta_{i+2}p_{i+2} + \delta_{i+1}p_{i+1} + t) \\
&Z_{i-1} = \alpha(\Delta + \delta_{k-1}p_{k-1} + \delta_{k-2}p_{k-2} + \cdots + \delta_{i+2}p_{i+2} + \delta_{i+1}p_{i+1} + t + p_i) \\
&\vdots \\
&Z_1 = \alpha(\Delta + \delta_{k-1}p_{k-1} + \delta_{k-2}p_{k-2} + \cdots + \delta_{i+2}p_{i+2} + \delta_{i+1}p_{i+1} + t + p_i + \cdots + p_2).
\end{aligned}$$

The tardiness cost associated with job j, $j = k, k+1, \dots, n$, is given by

$$\begin{aligned}
&Z_k = \beta(\delta_k p_k - \Delta) \\
&Z_{k+1} = \beta(\delta_k p_k - \Delta + \delta_{k+1}p_{k+1}) \\
&\vdots \\
&Z_n = \beta(\delta_k p_k - \Delta + \delta_{k+1}p_{k+1} + \cdots + \delta_n p_n).
\end{aligned}$$