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Stress testing correlation matrices for risk management

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ABSTRACT

Evaluating portfolio risk typically requires that correlation estimates of security returns be made. Historical financial events have shown that correlations can rise quickly, causing a huge increase in portfolio risk. Therefore, in stress testing portfolios, it is important to consider the influence of a sudden surge in selected correlations. Standard correlation stress testing mechanisms require us to change the selected correlations to designated values. However, the correlation matrix may become non-positive definite after some of its entries are altered. This paper proposes a blocking method by which an existing correlation matrix can be converted to incorporate change while keeping the matrix positive definite. In comparison with existing methods that usually only achieve semi-positive definiteness, the proposed method outperforms in the revised elements, while the approximation error of the non-revised elements is maintained within acceptable levels. Simulations show that our method is efficient and performs well for dimensions of 100, 500 and 1000. Our method is also shown to be more reliable in stress testing higher dimension correlation matrices. Information on the performance of the blocking method using a high-dimensional real data is also provided.

1. Introduction

Stress testing has become a major financial methodology in the risk management field, not only because it can assist financial institutions in understanding the effect of stress scenarios (Alexander

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& Sheedy, 2008; Berkowitz, 1999; BIS, 2000, 2005), but also because it can help with calculating risk measures that focus on extreme market conditions. For example, we may want to know how portfolio values change in response to a very abnormal directional movement in the component returns forming the portfolio. Stress scenarios are sometimes linked with high market variation, and thus can be referred to as situations in which there is an extraordinarily high level of volatility in financial securities. Practitioners and academics alike have devoted themselves to the investigation of how stress testing can be used to develop preventive actions or to determine regulatory capital. In more specific terms, we can determine stress-tested Value at Risk (VaR) for capital adequacy requirement calculations (Kupiec, 1998; Tan & Chan, 2003). In addition to the application to returns and volatility, stress testing correlations have also received much attention in risk management research. One major reason for this interest in correlations is that historical data provide substantial evidence of extreme levels of co-movement in financial returns during episodes of financial turmoil. Therefore, the correlation matrix of the underlying asset returns of a portfolio can change dramatically when a financial crisis occurs. In adverse situations, correlations can move to unexpectedly extreme levels either upwards or downwards. Bhansali and Wise (2001) described some cases in which correlations can become very large in stress situations. Kim and Finger (2000) used estimated conditional correlations for stress testing. By incorporating anticipated abnormal changes into the correlation matrix, one can derive the effects of stressed correlations on portfolio risk. More broadly, such effects can be translated into the regulatory capital required for risk management purposes.

One technical challenge of correlation stress testing is to ensure the mathematical validity of a stress correlation matrix (SCM). Specifically, we need to provide a positive-definite SCM, which is not trivial due to various reasons. This problem of converting a non-positive definite SCM, $\boldsymbol{\Gamma}^*$, into a positive definite SCM, $\tilde{\boldsymbol{\Gamma}}^*$, has been investigated since Finger (1997) proposed a simple approach in which new random variables are defined. However, this method is hard to control for elements in $\boldsymbol{\Gamma}^*$ we do not want to change. Kupiec (1998) suggested a shrinkage idea by which $\tilde{\boldsymbol{\Gamma}}^*$ is defined as a linear combination of $\boldsymbol{\Gamma}^*$ and a positive definite matrix. Clearly, assuming linearity requires justification, especially in high dimensions. Rebonato and Jäckel (2000) proposed a method for finding the $\tilde{\boldsymbol{\Gamma}}^*$ closest to $\boldsymbol{\Gamma}^*$ with respect to the Frobenius norm (Higham, 1988, 2002). Although their method is easy to implement, it only gives a positive semi-positive definite $\tilde{\boldsymbol{\Gamma}}^*$. Bhansali and Wise (2001) improved Robonato and Jäckel's (2000) approach, but their modification still provides only a positive-semi definite SCM. Turkay, Epperlein, and Christofides (2003) suggested a local eigenvalue modification that focuses on converting the part of $\boldsymbol{\Gamma}^*$ involving stressed correlations. Ndiaye, Oustry, and Piolle (2006) and Qi and Sun (2010) recently proposed convex optimizations to minimize the distance between $\boldsymbol{\Gamma}^*$ and $\tilde{\boldsymbol{\Gamma}}^*$ while maintaining $\tilde{\boldsymbol{\Gamma}}^*$ as positive definite.

This paper introduces a blocking method by which to convert an existing correlation matrix to incorporate stress scenarios while keeping the matrix positive definite. The blocking mechanism facilitates even high-dimensional applications with 1000 assets in the portfolio, and makes our algorithm very computationally efficient. An added advantage that is new to the literature is that it enables one to control the relative importance of correlations that do and do not need to be stressed.

The remainder of this paper is organized as follows. Section 2 reviews some existing methods. Section 3 presents our blocking method for correlation stress testing. Simulation studies are presented in Section 4, in which our method is shown to provide performance superior to that of existing methods. Section 5 reports a real data study using stock returns in Hong Kong. Section 6 sets out our conclusions.

2. Existing methods

In this section, we review two existing methods that utilize the spectral decomposition of correlation matrices. The use of eigenvalue-eigenvector analysis for correlation stress testing can be dated back to the study of Rebonato and Jäckel (2000). The main idea of their paper is to find the 'closest'