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# A trade-off result for preference revelation

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**Abstract**

If the social choice rule $g$ selects from one up to $k$ alternatives (but not more), then there exists a coalition $H$ of $k$ individuals such that for each profile $r$, the choice set $g(r)$ is the collection of the top-most alternatives in the orderings of the individuals in $H$. Consequently, $g$ is independent of the preferences of individuals not in $H$, forcing a disagreeable trade-off: Either some choice sets are very large, or most individuals never have any say in the social choice. 

*Keywords:* Trade-off; Social choice; Preference

## 1. Introduction

Gibbard (1973) and Satterthwaite (1975) have shown that, subject to a range condition, strategy-proofness implies dictatorship for *resolute* social choice procedures, i.e., for procedures where the choice set always contains just a single alternative. This paper characterizes social choice procedures that allow more than one alternative to be selected. There are two quite different reasons for relaxing resoluteness.

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First, although resoluteness might be quite desirable, non-dictatorship is even more desirable; and then we ask a trade-off question: If we relax resoluteness slightly, allowing small non-singleton choice sets, is there then a way to construct strategy-proof social choice rules that are far from dictatorial? Or is it the case that getting far from dictatorship forces some choice sets to be quite large?

Beyond this trade-off approach, which assumes that resoluteness is desirable, sometimes we actually want to chose more than one alternative. First, we might be interested in a social choice procedure that represents a preliminary stage in a process, say a piano competition, that ultimately chooses a singleton outcome, but it is actually desirable to have several alternatives selected at the preliminary stage.

Secondly, even where we are settling final outcomes, we may desire that a non-singleton set of alternatives be selected in some situations [2]: The International Mathematical Society will select up to four Fields Medalists to be announced at their next Congress. Imagine a mathematician asked to take part in the process of determining the recipients of the four 1998 Fields Medals. One *may* feel comfortable writing down a ranking of the say 25 candidates proposed but feel baffled by the request to rank the 12,650 quadruples of candidates. We will assume that individuals have a complete ordering of $X$ itself. An individual is assumed to be able to compare any *two* subsets of $X$, and the comparison will have to be consistent with the ordering of $X$ in the sense specified in the Section 2.

Section 2 provides the basic notation and definitions. In Section 3 we prove that the only strategy-proof rules that select from 1 to $k$ alternatives identify a set of $k$ individuals and then select the set of their top-most alternatives.

Ching and Zhou (1997) derive dictatorship from strategy-proofness, and without any condition on the size of the choice sets. Because the rules characterized by our theorem are not dictatorial (except when $k_{\max}(g) = 1$), the Ching–Zhou notion of strategy-proofness is very demanding when $g$ is not resolute. Using a less demanding definition of strategy-proofness, Duggan and Schwartz (1997) prove that there is an individual whose most-preferred alternative always belongs to the set of alternatives selected by the social choice function. Our theorem provides more information, but our strategy-proofness requirement is more exacting than that of Duggan and Schwartz.

Baigent (1998) examines strategy-proofness for non-resolute social choice rules, and in his case the domain is restricted to the set of dichotomous preferences — an alternative is either acceptable or not: $Y$ is preferred to $Z$ if every member of $Y$ is acceptable and some members of $Z$ are not acceptable, of if some members of $Y$ are acceptable and every member of $Z$ is unacceptable.

[2] A companion paper investigates cases where we *never* want a single alternative to be selected. (Campbell and Kelly, 1999) For example, exactly $k$ organs are available for transplant.