Extracting inflation expectations and inflation risk premia from the term structure: A joint model of the UK nominal and real yield curves

Michael A.S. Joyce [a,*], Peter Lildholdt [b], Steffen Sorensen [c]

[a] *Monetary Analysis, Bank of England, Threadneedle Street, London EC2R 8AH, United Kingdom*
[b] *Economic Analysis, Group Finance and Risk Management, A.P. Moller-Maersk Group, Denmark*
[c] *Financial Economic Research, Barrie and Hibbert Ltd., 41 Lothbury, London EC2R 7HG, United Kingdom*

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ABSTRACT

This paper analyses the UK interest rate term structure over the period since October 1992, when the United Kingdom adopted an explicit inflation target, using an affine term structure model estimated using both government bond yields and survey data. The model imposes no-arbitrage restrictions across nominal and real yields, which enables interest rates to be decomposed into expected real policy rates, expected inflation, real term premia and inflation risk premia. The model is used to shed light on major developments over the period, including the impact of Bank of England independence and the low real bond yield 'conundrum'.

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1. Introduction

The nominal and real interest rate term structures implied by UK government conventional and index-linked bonds can potentially provide monetary policy makers[1] with a great deal of information about financial market expectations of both future interest rates and inflation. The nominal and real term structures embody market expectations of future nominal and real interest rates respectively, while the difference between the two – the inflation term structure – embodies information about inflation expectations. Extracting this information, however, is complicated by the fact that the interest rate term structure may also reflect inflation risk premia and real term premia.[2]

The main contribution of this paper is to estimate a joint model of the UK nominal and real term structures over the period that the United Kingdom has had an explicit inflation target, enabling us to decompose nominal forward rates into expected real policy (risk-free) rates, expected inflation, real term premia and inflation risk premia since October 1992. Although we are not the first to do so, there are surprisingly few previous papers that have estimated theoretically consistent term structure models using UK data on both index-linked and nominal bond yields. One earlier example is a paper by Gong et al. (1998), but the generalised CIR (Cox et al., 1985) model specification that they adopt is very restrictive and has been shown to fit term structure data poorly. Evans (2003) estimates an extended Vasicek (1977) model that incorporates Markov-switching regimes. But while that model allows term and inflation risk premia to vary over time according to three regimes, this set-up is still rather restrictive. An unpublished paper by Risa (2001), applies a more flexible essentially affine model, similar to our own, to modelling UK data from 1983 to 1999, but does not incorporate survey information on inflation expectations as we do (see discussion below). Moreover, given his sample period, Risa does not shed much light on the impact of Bank of England independence on the term structure of interest rates and does not analyse the reasons for the period of unusually low long-term real interest rates – christened the bond yield 'conundrum' by Greenspan (2005) – that began in 2004.

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* Corresponding author. Tel.: +44 20 7601 3413; fax: +44 20 7601 5953.
  E-mail addresses: mike.joyce@bankofengland.co.uk (M.A.S. Joyce), plildholdt@gmail.com (P. Lildholdt), steffen.sorensen@barrhibb.com (S. Sorensen).

[1] Our focus is on monetary policy analysis, but of course information in the yield curve has a number of other uses; for example, it is important for risk management in financial institutions (see, e.g., Drehmann et al. (2009)).

[2] Government bond yields, particularly those for index-linked bonds, may also embody liquidity premia (the extra return investors require to compensate them for expected future illiquidity and illiquidity risk), but such premia are unlikely to have been a material influence on the UK term structure over most of our sample period and they are not explicitly incorporated into our modelling framework. We do, however, discuss the impact on our model estimates of specific episodes of extreme financial turbulence that may have affected liquidity premia, including the market disruptions that began in summer 2007 towards the end of our estimation period.

Our proposed model is based on the so-called essentially affine class of term structure models (see, e.g., Duffee (2002)). The approach has two main elements. First, we assume that UK nominal and real bond markets are arbitrage free, so that it is not possible to make risk-free profits from trading combinations of real and/or nominal bonds.[3] Second, we assume that bonds are priced by a stochastic discount factor (SDF) that takes a particularly flexible form, where the market price of risk is a linear function of the observable and unobservable factors in the model. A consequence of the second assumption, and the main implication of the essentially affine model, is that in our model bond prices, and thus yields, are linearly related to inflation and a small set of unobservable latent factors. We favour latent factors rather than macro-factors, partly because this approach has been shown to provide a better statistical fit of term structure data and partly because by taking an agnostic approach to the underlying factors driving yields the resulting model may be less prone to misspecification.[4]

We assume that two latent factors drive movements in expected real short rates and that the same two factors and two additional ones (one retail prices index (RPI) inflation, the other unobservable) drive the nominal curve and real term premia. An important feature of the model is that the same real SDF is assumed to price both real and nominal bond yields. To the extent that institutional investors have preferred habitats for index-linked bonds and demand/supply imbalances push prices away from fundamentals, this assumption may not be an accurate description of the real world. However, if the importance of demand/supply imbalances changes over time and is not a permanent feature, the ability of the model fit to various segments of the forward curve may enable us to identify the emergence of such non-fundamental or market segment-specific factors.

Joyce et al. (2008) apply a similar essentially affine model to the UK real term structure in isolation, in order to investigate the emergence of unusually low long real interest rates during 2004–05. One disadvantage of applying the affine modelling framework purely to real yields is the lack of available shorter maturity index-linked bonds. In this paper, by including nominal bond yields and inflation in the model, we are able to derive model-based estimates of short-maturity real rates. Moreover, estimating a joint model provides a number of advantages because it ensures consistency between the two term structures by imposing no arbitrage across them. And by modelling the dynamics of inflation expectations as a function of the information that drives real interest rates, as well as nominal rates and inflation, we use current information in the whole term structure to extract measures of expected inflation and inflation premia.

To reduce the possibility of encountering instability in term structure behaviour resulting from changes in the United Kingdom's monetary framework, we limit our sample to the period since October 1992, during which the United Kingdom has operated an inflation target.[5] However, this means there is potentially a small sample problem. As Kim and Orphanides (2005) demonstrate, small sample bias in term structure models can lead to implausible implications for the model-implied decompositions of forward rates into expected future short rates and term premia,

and in particular to low persistence in estimated expected short rates. In their application to the US yield curve, Kim and Orphanides (2005) advocate including survey data on the future path of interest rate expectations, as a way of supplementing the available time-series data on yields.[6] In our paper we incorporate bi-annual Consensus survey information on expected average inflation five to 10 years ahead, as an additional information variable, which helps to identify long-run inflation expectations. The implicit assumption is that the long-run inflation expectations of bond market participants will be the same as those of the economic and financial forecasters surveyed by Consensus forecasts. But we include an error term to allow long-term inflation expectations from the model to differ from those of the survey, so that expectations need only be the same on average over the sample. So, although our model incorporates survey information, the model forecasts are not always in line with the surveys. And, while the long-term survey information is only available every six months, our model has the advantage that it provides monthly estimates of expected inflation at any horizon.

The paper is structured as follows: Section 2 sets out the theoretical relationships between nominal and real yields and the SDF that obtain under no arbitrage and shows how we can use them to decompose interest rates into expected future short rates and premia. We also discuss the real SDF imposed in the essentially affine term structure literature. In Section 3, we discuss the econometric methodology used to estimate the model and derive the link between bond yields and the set of variables explaining yields. In Section 4 we describe the data used for our empirical analysis. Section 5 presents the results and Section 6 concludes. Some of the main mathematical derivations are explained in the Appendix at the end of the paper.

2. Theory and the EATS model

In this section we establish the relationship between nominal and real bond prices and the real SDF in the absence of arbitrage possibilities. We show how we can decompose nominal and real interest rates into expected future short interest rates and term premia and then how the difference between nominal and real interest rates, i.e. inflation breakeven rates, can be broken down into inflation expectations, inflation risk premia and an inflation convexity effect. Finally, we describe the modelling approach adopted in this paper, which is based on an essentially affine term structure (EATS) model.

2.1. The link between the nominal and real SDFs under no arbitrage

In an arbitrage-free environment, where all risk-free profit opportunities are eliminated, we can think of investors as pricing assets according to the fundamental asset pricing equation, i.e. by the discounted present value of their future pay-offs (for discussion on this see, e.g., Cochrane (2005)). So the current price of a zero-coupon real bond, denoted by $P_t^{n,R}$, that pays one unit of the consumption good when it matures in period $t + n$ is given by

$$P_t^{n,R} = E_t[M_{t+1} \cdot M_{t+2} \cdots M_{t+n}], \tag{1}$$

where M_{t+j} denotes the real SDF in period j. This no-arbitrage condition corresponds to the Euler condition in a representative agent

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[3] As we show below, the assumption of no arbitrage enables us to decompose interest rates into expectations of future risk free rates and term premia. But, as well as being theoretically attractive, there is some evidence that imposing no-arbitrage restrictions significantly improves out-of-sample interest rate forecasts (see, e.g., Almeida and Vicente (2008)).

[4] Most macro-factor models of the term structure find it necessary to include additional latent factors to fit longer-maturity yields (see, e.g., Ang and Piazzesi (2003)).

[5] Using a long run of data covering the period from 1881 to 2003, Kanas (2008) finds that regime switching in the dynamics of US and UK real interest rates is driven by changes in inflation and political regimes.

[6] In a subsequent paper, Kim and Wright (2005) incorporate survey information on both expected policy rates and inflation into their model of the US nominal and real term structures, but they do not incorporate market data on US index-linked bonds, so-called Treasury Inflation Protected Securities (TIPS), because of the lack of a long enough back run of data. More recently, D'Amico et al. (2008) have attempted to estimate a model using TIPS data. However, the sample available to them is still quite short (only 9 years) and TIPS appear to contain a sizable liquidity premium until quite recently.