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Inventory and pricing strategies for deteriorating items with shortages: A discounted cash flow approach

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Abstract

In this article, we consider an infinite horizon, single product economic order quantity where demand and deterioration rate are continuous and differentiable function of price and time, respectively. In addition, we allow for shortages and completely backlogged. The objective is to find the optimal inventory and pricing strategies maximizing the net present value of total profit over the infinite horizon. For any given selling price, we first prove that the optimal replenishment schedule not only exists but is unique. Next, we show that the total profit per unit time is a concave function of price when the replenishment schedule is given. We then provide a simple algorithm to find the optimal selling price and replenishment schedule for the proposed model. Finally, we use a couple of numerical examples to illustrate the algorithm.

Keywords: EOQ; Pricing; Time-value of money; Deterioration; Shortages

1. Introduction

In many inventory systems, the deterioration of goods is a realistic phenomenon. It is well known that certain products such as medicine, volatile liquids, blood bank, food stuff and many others, decrease under deterioration (vaporization, damage, spoilage, dryness and so on) during their normal storage period. As a result, while determining the optimal inventory policy of that type of products, the loss due to deterioration can not be ignored. In the literature of inventory theory, the deteriorating inventory models have been continually modified so as to accommodate more practical features of the real inventory systems. The analysis of deteriorating inventory began with Ghare and Schrader (1963), who established the classical no-shortage inventory model with a constant rate of decay. However, it has been empirically observed that failure and life expectancy of many items can be expressed in items of Weibull distribution. This empirical observation has prompted researchers to represent the time to deterioration of a product by a Weibull distribution. Covert and Philip (1973) extended Ghare and Schrader's (1963) model and obtained an economic order quantity model for a

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variable rate of deterioration by assuming a two-parameter Weibull distribution. Researchers including Philip (1974), Misra (1975), Tadikamalla (1978), Chakrabarty, Giri, and Chaudhuri (1998) developed economic order quantity models which focused on this type of products. Therefore, a realistic model is the one that treats the deterioration rate as a time varying function. Some models have been proposed for more information, we refer the reader to Ghare and Schrader (1963) and the references therein.

Pricing is a major strategy for a seller to achieve its maximum profit. Consequently, several researchers in operations management have studied the joint lot sizing and pricing decisions for deteriorating items. Cohen (1977) jointly determined the optimal replenishment cycle and price for inventory that is subject to continuous decay over time at a constant rate. Wee (1997, 1999) extended Cohen's model to consider a Weibull distribution deterioration item with shortage. Then, Wee and Law (2001) extended Wee's (1997) model and applied the DCF (Discounted Cash Flow) approach to the finite planning horizon in which the replenishment cycle is known. All the above models assumed a linear form of the price-dependent demand rate. Recently, Hwang and Shinn (1997) addressed the joined price and lot size determination problem for an exponentially deteriorating product and iso-elastic demand when the vendor permits delay in payments. Mukhopadhyay, Mukherjee, and Chaudhuri (2004, 2005) re-established Cohen's model (1977) by taking iso-elastic demand and a varying deterioration rate. However, it is very restrictive to assume that the item deteriorates at a specific distribution and the demand follows a specific function. To relax these assumptions, Abad (1996, 2001) discussed a lot-sizing problem for a product with a general deterioration function and a general demand function, allowing shortages and partial backlogging. Unfortunately, he does not use the stockout cost (includes backorder cost and the lost sale cost) in the formulation of the objective function since these costs are not easy to estimate, and its immediate impact is that there is a lower service level to customers.

Companies have recognized that besides maximizing profit, customer satisfaction plays an important role for getting and keeping a successful position in a competitive market. The proper inventory level should be set based on the relationship between the investment in inventory and the service level. For inventory systems, the average cost approach is more frequently used by the practitioners when the discount rate is at a negligible level. However, as the time value of money is taken into account in the inventory systems, an alternative is to determine the decision variables by minimizing the discounted value of all future costs (i.e., the net present value (NPV) of total cost). Hadley (1964) compared the optimal order quantities determined by minimizing these two different objective functions. When the discount rate is excessive, he obtained the optimal reorder intervals with significant differences for these two models. Further, Rachamadugu (1988) developed error bounds for EOQ model by minimizing net present value. Since the net present value is the standard methodology in theoretical analysis and the most frequently used method for making financial decisions, we develop a generalized inventory model using the net present value of its total profit as the objective function to amend the papers of Cohen (1977), Wee (1997), Wee and Law (2001), Abad (1996, 2001) and Mukhopadhyay et al. (2004, 2005) with a view to making the model more relevant and applicable in practice. In the next section, the assumptions and notation related to this study are presented. Then, we prove that the optimal replenishment policy not only exists but is unique, for any given selling price. Next, we show that the net present value of total profit is a concave function of selling price when the replenishment schedule is given. We also provide a simple algorithm to find the optimal replenishment schedule and selling price for the proposed model. Finally, we use a couple of numerical examples to illustrate the procedure of solving the model.

2. Model notation and assumptions

To develop the mathematical model of inventory replenishment policy, the notation adopted in this paper is as below:

A = the replenishment cost per order
c = the purchasing cost per unit
s = the selling price per unit, where $s > c$
c_1 = the holding cost per unit time
c_2 = the backorder cost per unit time
r = the discount rate