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The distribution-free newsboy problem: Extensions to the shortage penalty case

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Abstract

In the classical newsboy problem, no cost is assumed if the ordered quantity is less than the demand. However, in reality failure to meet demand is always associated with a penalty. The aim of this work is to extend the analysis of the distribution-free newsboy problem to the case when shortage cost is taken into consideration. The analysis is based on the assumption that only the mean and variance of demand are known, but its particular probability distribution is not. A model is presented for determining both an optimal order quantity and a lower bound on the profit under the worst possible distribution of the demand. The following cases are considered: the single product case, the fixed ordering cost case, the random yield case, and the resource-constrained multi-product case.

Keywords: Newsboy problem; Distribution free approach; Single-period inventory systems; Lot sizing

1. Introduction

The classical newsboy problem aims to find the order quantity for a given product that maximizes the expected profit in a single period, probabilistic demand framework. Gallego and Moon (1993) define the newsboy problem as the tool to decide the stock quantity of an item when there is a single purchasing opportunity before the start of the selling period, and the demand for the item is random. The classical newsboy model assumes that if the order quantity is larger than the realized demand, a single discount is used to sell excess inventory or that excess inventory is disposed off. On the other hand, if the order quantity is less than demand, then profit is lost. The objective is to find the optimum tradeoff between the risk of overstocking (incurring disposal cost) and the risk of under stocking (losing profit).

The newsboy problem is a classical inventory problem that is very significant in terms of both theoretical and practical considerations. Items that can be classified under single-period inventory systems include

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Christmas trees, new-year greeting cards, and of course daily newspapers. The newsboy problem is often used to aid decision-making in fashion, sporting industry, and apparel industry, both at the manufacturing and retailer level (Gallego and Moon, 1993). According to Weatherford and Pfeifer (1994), newsboy models are also used in capacity management and booking decisions in service industries such as hotels and airlines.

Several models and extensions of the classical newsboy problem have been proposed and solved in the literature. Khouja (1999) classifies solution approaches into two types; the first minimizes expected cost, while the second maximizes expected profit. Khouja (1999) also classifies extensions to the basic newsboy problem into 11 categories, including: random yields where orders received may include defective items, different states of information about demand, and resource-constrained multiple products. This paper addresses these three extensions, in addition to the fixed ordering cost case. However, the main focus of this paper in on the extension based on limited information about demand, namely the distribution-free newsboy problem.

Several authors have analyzed the distribution-free newsboy problem, in which the distribution of demand is not known but only the mean μ and variance σ^2 are specified. Usually, a minimax approach is followed, which aims to minimize the maximum cost resulting from the worst possible demand distribution. Scarf (1958), who pioneered this approach, uses it to develop a closed form expression for the order quantity that maximizes expected profit. Kasugai and Kasegai (1960, 1961), respectively, apply dynamic programming and the minimax regret ordering policy to the distribution-free multi-period newsboy problem.

Gallego and Moon (1993) provide a simpler proof of optimality of Scarf's ordering rule and extend the analysis to the cases of random yields, fixed ordering cost, and constrained multiple products, in addition to the recourse case where there is a second ordering opportunity. Moon and Choi (1995) extend the model of Gallego and Moon (1993) to the case in which customers may balk if the inventory level is low. Moon and Choi (1997) subsequently use a similar approach to analyze a newsboy problem with various degrees of product processing, from raw materials to the finished product. The alternative policies in this case include make-to-order, make-in-advance, and composite policies, with and without budget limitations.

Vairaktarakis (2000) develops several minimax regret models for the distribution-free multi-item newsboy problem under a budget constraint and two types of demand uncertainty. The interval type specifies a lower bound and an upper bound on demand, while the discrete type states a set of likely demand values. Moon and Silver (2000) develop distribution-free models and heuristics for a multi-item newsboy problem with a budget constraint and fixed ordering costs. Recent and comprehensive literature reviews and suggestions for future research on the newsboy problem are complied by Khouja (1999), Petruzzi and Dada (1999), and Silver et al. (1998).

The model of Gallego and Moon (1993), which constitutes the core background of this work, provides the optimal order quantity that maximizes the expected profit against the worst possible distribution of the demand with mean μ and variance σ^2. It also presents a simple lower bound on the expected profit with respect to all possible distributions of demand. The purpose of this work is to extend the optimal order quantity formulas found by Gallego and Moon (1993) to the case where shortage cost (above and beyond lost profit) is considered for the following cases: the single product case, the fixed ordering cost case, the random yield case, and the multi-product case. Since the recourse case considered by Gallego and Moon (1993) rules out shortages, it is not considered in this paper. Although the shortage (lost sales) cost is generally not easy to estimate, it is nonetheless a real cost that should not be simply ignored. Including any reasonable estimate of this cost significantly improves the accuracy and profitability of the newsboy model. Numerical experiments are presented to demonstrate the profit increase obtained from the new models that incorporate shortage cost.

This paper is organized as follows. In Section 2, the optimum order quantity and the lower bound on expected profit are derived for the single product case. The fixed ordering cost case is analyzed in Section 3,