Available online at www.sciencedirect.com
SCIENCE DIRECT
Int. J. Production Economics 97 (2005) 296–307
international journal of production economics
www.elsevier.com/locate/ijpe

An analysis of the multi-product newsboy problem with a budget constraint

Layek L. Abdel-Malek[a,*], Roberto Montanari[b]

[a]Department of Industrial and Manufacturing Engineering, New Jersey Institute of Technology, Newark, NJ 07102, USA
[b]Dipartimento di Ingegneria Industriale, Università degli Studi di Parma, Viale delle Scienze, 43100 Parma, Italy

Received 1 March 2004; accepted 1 August 2004
Available online 6 November 2004

Abstract

This is a sequel of an earlier paper entitled "Exact, approximate, and generic iterative models for the multi-product newsboy problem with budget constraint" (Abdel-Malek et al., 2004) that appeared in this journal. Motivated by Lau and Lau's (Eur. J. Oper. Res., 94 (1996) 29) observation where infeasible ordering quantities (negative) were obtained when applying existing methods, the extension here examines the solution space of the problem in order to provide the necessary insight into this phenomenon. The resulting insight shows that the solution space can be divided into three regions that are marked by two distinct thresholds. The first region is where the budget is large and the solution is the same as the unconstrained problem. The second region is where the budget is medium and the constraint is binding, however the newsboy can order all the products on the list. The third region is where the budget is very tight and if the non-negativity constraints are relaxed negative order quantities may be obtained, and therefore some products have to be deleted from the original list. We show how the values of the thresholds that divide the regions are computed and extend the previous methods, when necessary, to cover each of the three-solution's domains in order to determine the optimum order quantity for the various products. Numerical examples are given to illustrate the application of the developed procedures.

Keywords: Inventory control; Newsboy problem; Nonlinear optimization; Budget constraint

1. Introduction

The newsboy problem, also known as the single period stochastic inventory model, is found to be a suitable tool for decision-making regarding stocking issues in today's supply chains. Motivated by the interest of the community, in an earlier publication entitled "Exact, approximate, and generic iterative models for the multi-product newsboy problem with budget constraint" (Abdel-Malek et al., 2004), we developed models to determine the optimum lot size for the

*Corresponding author. Tel.: +1 973 596 3648; fax: +1 973 596 3652.
E-mail address: malek@njit.edu (L.L. Abdel-Malek).

0925-5273/$ - see front matter
doi:10.1016/j.ijpe.2004.08.008

capacitated situation. The model formulation used is as follows:

$$\text{Minimize } E = \sum_{\tau=1}^{N} \left[c_\tau x_\tau + h_\tau \int_0^{x_\tau} (x_\tau - D_\tau) f_\tau(D_\tau) \times \mathrm{d}D_\tau + v_\tau \int_{x_\tau}^{\infty} (D_\tau - x_\tau) \times f_\tau(D_\tau)\,\mathrm{d}D_\tau \right], \tag{1}$$

subject to

$$\sum_{\tau=1}^{N} c_\tau x_\tau \leqslant B_G, \tag{2}$$

where E is the expected cost; N the total number of items; τ the item index; c_τ the cost per unit of product τ; x_τ the amount to be ordered of item τ which is a decision variable; h_τ the cost incurred per item τ for leftovers at the end of the specified period; D_τ the random variable of item's τ demand; $f_\tau(D_\tau)$ probability density function of demand for item τ; v_τ the cost of revenue loss per unit of product τ, and B_G the available budget.

This model is based on the classical one that was developed originally by Hadley and Whitin (1963). The solution is obtained using the Lagrangian method and it is as follows:

$$F(x_\tau^{**}) = \frac{v_\tau - (1+\lambda)c_\tau}{v_\tau + h_\tau}, \tag{3}$$

where x_τ^{**} is the optimal solution under budget constraint; $F(\cdot)$ the cumulative distribution function (CDF), and λ the Lagrangian multiplier.

As can be seen, this approach first ignores the budget constraint and finds the optimum values of the lot size for each product. Then, these values are plugged into the budget to see if they satisfy its constraint, otherwise the inequality constraint is set to equality and the Lagrangian approach is used. It should be also noted that Hadley and Whitin's model relaxes the non-negativity constraints of the order quantities. In fact most of the existing Lagrangian based models regarding the capacitated newsboy problem do not pay too much attention to the lower bounds of the order quantities (non-negativity constraints), see for example, Abdel-Malek et al. (2004), Ben-Daya and Abdul (1993), Erlebacher (2000), Gallego and Moon (1993), Khouja (1999), Moon and Silver (2000), and Vairaktarakis (2000). One should mention that if, when tackling a three-product problem, the non-negativity constraints are not relaxed and Kuhn–Tucker conditions are applied, the number of nonlinear equations to be solved simultaneously is more than 20. This could be one of the reasons that most existing models relax the lower bounds to make the problem tractable. Nevertheless, in doing so and as Lau and Lau (1995, 1996) were among the first to observe, this could lead to infeasible order quantities (negative) for some of the considered products. While modeling a case study for a large bakery, Lau and Lau experienced this phenomenon and offered a conceptual explanation for its occurrence.

To address the aforementioned situation, we extend the previous paper by Abdel-Malek et al. to efficiently solve the capacitated multi-product newsboy problem (CMPNP) and to help the decision-maker in recognizing the implications of the available budget. Hence, the decision-maker can avoid infeasible (negative) order quantities by deleting products from further consideration when the constraint is too tight. Additionally, this extension provides a means to conduct sensitivity analysis, when necessary, for increasing the budget to include desired products if the initial amount does not allow for their consideration.

Our taxonomy in this article is as follows. After this introduction, in Section 2, we present the problem and give some insights into its solution space. Section 3 follows where a general approach for solving the CMPNP and the necessary proofs are shown. Afterwards, Section 4 shows numerical examples to illustrate the application of these methods. Finally, Section 5 presents the conclusion of this paper.

2. The problem

As mentioned in the introduction many of the existing methodologies for solving the newsboy problem with budget constraint have used the Lagrangian approach that was originally implemented by Hadley and Whitin (1963) as the underpinning for further developments. The problem with