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# The capacitated newsboy problem with random yield: The Gardener Problem

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ABSTRACT

This publication is the third in a series of articles by Abdel-Malek et al. [2004. Exact, approximate, and generic iterative models for the multi-product newsboy with budget constraint. International Journal of Production Economics 91, 189–198] and Abdel-Malek and Montanari [2005. An analysis of multi-product newsboy problem with a budget constraint. International Journal of Production Economics 97, 296–307], which appeared in IJPE addressing issues regarding the newsboy models. In this paper, models are developed to extend the existing ones to cover random yield scenarios. We designate this type of models as the Gardener Problem. The models are based on the application of Lagrange multipliers, Leibniz's rule and Newton's method to obtain the optimum solution for the considered random yield and probabilistic demand situations. The developed methodologies are applicable to general probability distribution functions. Examples are given to show the applicability of the developed approach to different probability distributions for both the supply and the demand as well as for interdependent yield structures.



## 1. Introduction

In today's global economy, we have become increasingly dependent on complex and intricate supply chains in the various industrial sectors. A key factor in supply chain management is inventory control. The newsboy model introduced by Hadley and Whitin has proven to be one of the most important tools in managing inventories in the present environment, see Denardo (2001). It is known that the newsboy dilemma is in ordering the optimum number of copies of a newspaper or a collection of them for the next day's distribution in order to minimize cost or maximize revenue. In some instances, the newsboy is bound by some constraints such as budget, and space. In Hadley and Whitin's (1963) models as well as in most extensions that followed, it is assumed that the newsboy gets the exact amounts ordered. Nevertheless, in other situations that is not the case. More clearly, the amounts received may vary from those ordered. Also, as in the original newsboy model the demand for the product is random. We name this extension as the Gardener Problem.

We consider a gardener who would like to divide available acreage among a number of possible crops for the coming season. The demand for each of the crops is random and the yield depends on the weather conditions of the season. The gardener's objective is to optimize his planting strategy and divide the available land (resource) among a list of possible crops. There are many similar situations in practice to that of the gardener. Among them are, these encountered in supply chains, particularly those of global nature where the buyers may opt for e-bidding as opposed to long-term partnerships. In the former situation, the buyer does not have knowledge of the product's quality that is received compared with the latter when the buyer knows with some certainty the percentage

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of defective in the items from the dedicated vendor and accordingly the quantity ordered can be adjusted. Among other comparable situations to that of the gardener are; (a) wafers' production where it is known that the yield and the demand are random (several earlier publications addressed this problem, but most of their focus is on the randomness of the production process and not on the demand), (b) the release of new video games before major Holidays when the retailer does not usually gets the amount ordered while still subject to random demands of these items, (c) a manufacturer who has to depend on recyclable materials for his production, although the tonnage may be what was asked for, the conversion rate from the supplied materials to the substances needed for manufacturing the product could vary from delivery to delivery due to the random nature of the disposables and the fact the demand for each of these recyclables can also vary, and (d) the production of vaccines before a flue season where the pharmaceutical manufacturer has to allocate the available production capacity beforehand among the various drugs while knowing neither the exact nature of the incoming season's particulars of the flue string and the yield of its production process, nor precisely the demand for them.

It is noteworthy to mention that random yields can occur in spite of total quality management, TQM, practices. This could occur in situations in which the inspection costs are prohibitively high and one cannot examine all items. Therefore, defects can be undetected from time to time; see Grosfeld-Nir et al. (2000).

As one can see from the foregoing paragraphs the wide range of situations encountered today where the Gardener Problem can be applied. Nevertheless, despite the numerous articles that appeared tackling the classical newsboy problem and many of its extensions, as will be seen later in Section 2, and the variety of applications where decision makers have to allocate their limited resources subject to randomness in both supply and demand, insufficient attention has been given to developing solution methodologies for solving the capacitated random yield case. Taking a step towards addressing the need for models in this arena is what motivated our development of the work here. More specifically, we will develop models and solution methodologies that consider different probability distributions for the yield (supply) and the traditional randomness in the demand as well as the degrees of tightness of the constraint, i.e. when the constraint becomes relatively tight or too tight to the extent that one or more product should be deleted from the original list (otherwise the traditional Lagrange multiplier approach may produce trivial solutions, i.e. negative order quantity). Additionally, we will provide numerical examples to illustrate the application of the various models as well as to show how to extend the developed models to situations where the random yield structure of the products is interdependent.

Our taxonomy in this work is as follows. We begin by a section where we review the pertinent literature (Section 2). Then two sections succeed. In Section 3, we present the model and introduce its necessary preliminaries and in Section 4, we illustrate the application of the developed models by giving numerical examples that cover different scenarios of randomness in supply and demand as well as several degrees of the constraint tightness and yield interdependencies. Finally, we conclude in Section 5.

## 2. Literature review

As mentioned in Section 1, there are many extensions to the classical newsboy problem as it was originally reported by Hadley and Whitin (1963). Nevertheless, despite the importance of random yield modeling in today's global environment, the literature in this area remains disproportional to such models' wide range of potential applications. In this section, we review some of the relevant literature regarding the current state-of-the-art of the newsboy problem and its solution methodology. Then we will follow by referring to pertinent articles that consider random yield issues.

We first refer to two reviews by Gallego and Moon (1993) and Khouja (1999). Both papers present comprehensive literature review of the newsboy problem and offer suggestions for extensions. One of the proposed extensions that are suggested by Khouja is that concerning the need for further development in the modelling of the newsboy problem with random yield (which is the intent of our work here). After these reviews, numerous articles have appeared addressing some of these extensions. Most of these articles have focused on developing solution methodologies for the multi-product newsboy model, introducing distribution-free approaches to estimate lot size when there is not enough information about the demand, or when returns of the product or mid period orders are available options to the buyer. In the following paragraphs, we briefly refer to some of these works.

Lau and Lau are credited for many developments in the newsboy problem formulation, application and solution methods. Among their accomplishments are those in applying the newsboy concept in defining strategic policies for return of seasonal products, developing simple solution methods for the multi-constraint newsboy model, and recognizing the inherent problem in solving the capacitated newsboy model when the budget constrain is tight (this could lead to negative order quantities of some products, trivial solution). The interested reader is referred to Lau and Lau (1995, 1996), and Lau et al. (2000) for further details. Another extension is attributed to Moon and Silver (2000) where they added fixed ordering costs to the classical newsboy with budget constraint. In the arena of product returns, Mostard et al. (2005) analyzed the distribution-free newsboy problem with returns, in which only the mean and variance of demand are assumed to be known. They derived a closed-form expression for the distribution-free order quantity, which they compared with the optimal order quantities when gross demand is assumed to be normal, lognormal or uniform.

Several solution methodologies have emerged addressing the capacitated multi-product newsboy problem. For example, the articles by Ben-Daya and Raouf (1993), and Erlebacher (2000) introduce optimal and heuristic