Int. J. Production Economics 129 (2011) 169–177

Contents lists available at ScienceDirect

Int. J. Production Economics

journal homepage: www.elsevier.com/locate/ijpe

Analysis of the newsboy problem with fuzzy demands and incremental discounts

Shih-Pin Chen*, Yann-Horng Ho

Department of Business Administration, National Chung Cheng University, Chia-Yi 621, Taiwan

ARTICLE INFO

Article history:
Received 13 December 2008
Accepted 17 August 2010
Available online 24 September 2010

Keywords:
Fuzzy sets
Inventory
Incremental discounts
Newsboy problem
Single-period problem

ABSTRACT

This paper proposes an analysis method for the single-period (newsboy) inventory problem with fuzzy demands and incremental quantity discounts. In fuzzy environments, the availability of the quantity discount makes the analysis of the associated model more complex. The proposed analysis method is based on ranking fuzzy number and optimization theory. By applying the Yager ranking method, the fuzzy total cost functions with different unit purchasing costs are transformed into convex piecewise nonlinear functions. To effectively and efficiently find the optimal inventory policy, the proofs of two properties regarding the relative position between the price break and minimums of these nonlinear functions are proposed. The closed-form solutions to the optimal order quantities are also derived. Four cases of a numerical example are solved to demonstrate the validity of the proposed analysis method. It is clear that the proposed methodology is applicable to further cases with different types of quantity discounts and other more complicated cases. More importantly, managerial implications are also provided for decision-makers' references.

1. Introduction

Effective and efficient inventory management is becoming increasingly important for all types of organizations and their supply chains in today's highly competitive business environment (Arshinder et al., 2008; Krajewski et al., 2010; Stevenson, 2005). One type of inventory problem often encountered with seasonal or customized products is the newsboy problem, also called the single-period inventory problem or the newsvendor problem (see, e.g., Mostard et al., 2005; Wu et al., 2007; Serel, 2008; Özler et al., 2009), since only a single procurement is made (Hadley and Whitin, 1963; Tersine, 1988; Hillier and Lieberman, 2005). A typical example is making a one-period decision regarding how many items of a seasonal good, which cannot be sold the next year, to purchase for the current year.

One of the principal factors affecting decisions on order quantity in the newsboy problem is the nature of the demand. In real life, demands are uncertain and need to be estimated from historical data and described by certain probability distributions under traditional (crisp) environments. However, there are cases where the probability distribution of the demand cannot be obtained. For example, the probability distribution of the demand of new products is usually unknown due to lack of historical information. In this case the demands are suitably described subjectively by linguistic terms, such as "high", "low", or "approximately equal to a certain amount." That is, the demands are fuzzy rather than probabilistic. Fuzzy set theory has been applied to inventory problems, with demand uncertainties attributed to fuzziness rather than randomness (see, e.g., Park, 1987; Roy and Maiti, 1997; Chang and Yao, 1998; Lee and Yao, 1998; Yao and Lee, 1996, 1999; Chang, 1999; Amid et al., 2009; Shi and Zhang, 2010). Additional related studies can be found in Mula et al. (2006) which provides a good review. Several scholars have approached the newsboy problem from the standpoint of fuzzy environments. For example, Petrovic et al. (1996) proposed a model for the newsvendor problem where the demand is described by a discrete fuzzy set in which the cost is presented by a triangular fuzzy number. Ishii and Konno (1998) used the fuzziness concept to consider shortage cost in the classical newsvendor problem, although the demand was still stochastic. Kao and Hsu (2002) proposed a model to find the optimal order quantity of the classical single-period problem with fuzzy demand. Li et al. (2002) proposed two models for newsvendor problems that have two types of uncertainties, one of which is randomness and the other is fuzziness.

Quantity discounts often occur in practice, though they have not been discussed in the aforementioned articles. In fact, only a few articles have been published on the fuzzy inventory problem with quantity discounts. Lam and Wong (1996) applied fuzzy mathematical programming to solve economic lot-size problems with multiple price breaks, though they did not focus on newsboy problems. On the basis of genetic algorithms and fuzzy

* Corresponding author. Tel.: +886 5 2720411; fax: +886 5 2720564.
E-mail address: chensp@ccu.edu.tw (S.-P. Chen).

0925-5273/$ - see front matter © 2010 Elsevier B.V. All rights reserved.
doi:10.1016/j.ijpe.2010.09.014

simulation, Ji and Shao (2006) proposed a hybrid intelligent algorithm to solve the bi-level newsboy problem with fuzzy demand and discounts, but they did not provide analytical solutions.

This paper considers the newsboy problem with incremental quantity discounts and demands being fuzzy numbers, with the objective of minimizing the total cost per unit time. Next, a solution procedure for finding the optimal inventory policy is developed, and the results are illustrated with numerical examples. Finally, discussions and conclusions together are presented.

2. Fuzzy numbers and fuzzy representation

Fuzzy set theory, introduced by Zadeh (1965), has a wide range of applications within many areas, such as management, engineering, medicine, computer, life, and social sciences (Zimmermann, 2001). Its application is so extensive since many decision-making tasks are too complicated to be described quantitatively in a precise manner. In this case, fuzzy set theory has been demonstrated to be a proper means for modeling imprecision or uncertainty arising from the situations of human reasoning or mental phenomena. In brief, a fuzzy set is a generalization of the classical (or crisp) set, and it is a class of objects with membership grades defined by a membership function. For example, a fuzzy set $\tilde{A}$ can be mathematically described by $\tilde{A}=\{x\in X|\mu_{\tilde{A}}(x)\}$, where X is the universal set and $\mu_{\tilde{A}}(x)$ is the membership function. An α-cut (or α-level set, $\alpha\in[0,1]$) of the fuzzy set $\tilde{A}$ is a crisp set $A(\alpha)$ that contains all the elements of the universal set X that have a membership grade in $\tilde{A}$ larger than or equal to the specific value of α: $A(\alpha)=\{x\in X|\mu_{\tilde{A}}(x)\geq\alpha\}$. Specifically, a fuzzy number is a convex and normalized fuzzy set whose membership function is piecewise continuous, in that a fuzzy set is convex if and only if each of its α-cuts is a convex set; a fuzzy set is called normalized when at least one of its elements attains the maximum possible membership grade (Klir and Folger, 1992).

The following realistic example shows how firms actually use fuzzy numbers to describe their imprecise demands. Chia-Yi City is one of the larger cities in the southwest part of Taiwan, close to the Tropic of Cancer 22.5°N. One firm produces a new type of Christmas decoration in Chia-Yi City, due to lack of historical information, the probability distribution of the demand for this new product is unknown. Thus, the demands are imprecise and can be only linguistically estimated by the marketing manager: "The demand in this Christmas season will most likely be between 1400 and 1600 units; not less than 1000 units nor more than 2000 units." Clearly, this demand is more suitably described by a fuzzy number than a crisp one. The marketing manager subjectively described the fuzzy demand as a trapezoidal fuzzy number [1000, 1400, 1600, 2000] whose membership function is as follows:

$$\mu(x)=\begin{cases} L(x)=(x-1000)/400, & 1000\leq x\leq 1400,\\ 1, & 1400\leq x\leq 1600,\\ R(x)=(2000-x)/400, & 1600\leq x\leq 2000,\end{cases}$$

where $L(x)$ and $R(x)$ are reference functions. Note that here they are set as linear functions according to the marketing manager's subjective judgement; in fact, they can be nonlinear. It is clear that the question of how the marketing manager chooses the reference functions is important. This question has been studied by other papers and several solutions have been proposed (e.g., Turksen, 1991).

3. Fuzzy single-period model with incremental quantity discount

Consider a single-period inventory problem with fuzzy demand and incremental quantity discounts. The demand is subjectively determined as a fuzzy number. The supplier offers his product at lower prices to those who buy them in large quantities. In general, an incremental quantity discount schedule with k discounts for the product can be represented as follows:

$$c(Q)=\begin{cases} c_0, & \text{if} \quad Q^s_{c_0}\leq Q<Q^s_{c_1},\\ c_1, & \text{if} \quad Q^s_{c_1}\leq Q<Q^s_{c_2},\\ \vdots & \qquad \vdots\\ c_k, & \text{if} \quad Q^s_{c_k}\leq Q,\end{cases} \tag{1}$$

where $c_0>c_1>\cdots>c_k$, c_j is the unit purchasing price if the order quantity Q falls in the $(j+1)$st discount interval of $[Q^s_{c_j},Q^s_{c_{j+1}})$, $Q^s_{c_j}$ and $Q^s_{c_{j+1}}$ are quantities that define discount intervals, $j=0$, 1, …, k, and $Q^s_{c_{k+1}}$ represents infinity. Note that, in contrast to a total discount system, in an incremental discount system the lowest price is paid only for units in the highest interval and there are higher prices for quantities in lower intervals.

Without loss of generality, consider the simplest case of one incremental discount, Q^s_c. The cases of multiple incremental discounts can be generalized from this case. Suppose that the fuzzy demand is a trapezoidal fuzzy number $\tilde{\lambda}=[l, m, n, u]$ described by the following membership function:

$$\mu(x)=\begin{cases} L(x)=(x-l)/(m-l), & l\leq x\leq m,\\ 1, & m\leq x\leq n,\\ R(x)=(u-x)/(u-n), & n\leq x\leq u.\end{cases} \tag{2}$$

The problem is to find the optimal order quantity, Q^*_D, such that the fuzzy total cost is minimized:

$$\tilde{T}(Q)=\begin{cases} \tilde{T}_0(Q)=c_0Q+p\max\{0,\tilde{\lambda}-Q\}+h\max\{0,Q-\tilde{\lambda}\}, & \text{if } Q<Q^s_c,\\ \tilde{T}_1(Q)=c_0Q^s_c+c_1(Q-Q^s_c)+p\max\{0,\tilde{\lambda}-Q\}+h\max\{0,Q-\tilde{\lambda}\}, & \text{if } Q\geq Q^s_c.\end{cases} \tag{3}$$

where p is the selling price per unit ($p>c_0>c_1$), h is the holding cost per unit at the end of period ($h<0$, representing the salvage value per unit), and generally the salvage value is less than the unit cost, i.e., $c_0>c_1>-h$ (Kao and Hsu, 2002).

Clearly, the optimal order quantity of the fuzzy single-period inventory model, denoted as Q^*_D, will occur between l and u since $p>c_0>c_1$ and $c_0>c_1>-h$ (Kao and Hsu, 2002). It is clear that this property also holds for the proposed model according to Eq. (3). If the fuzzy demand degenerates as a crisp value of x, then for a prespecified order quantity Q, the total cost (3) depends on x and becomes

$$T(x|Q)=\begin{cases} c_0Q+h(Q-x), & \text{if } l\leq x\leq Q<Q^s_c,\\ c_0Q^s_c+c_1(Q-Q^s_c)+h(Q-x), & \text{if } l\leq x\leq Q \text{ and } Q\geq Q^s_c,\\ c_0Q+p(x-Q), & \text{if } Q\leq x\leq u \text{ and } Q<Q^s_c,\\ c_0Q^s_c+c_1(Q-Q^s_c)+p(x-Q), & \text{if } Q^s_c\leq Q\leq x\leq u.\end{cases} \tag{4}$$

Therefore, on the basis of Zadeh's extension principle (Zimmermann, 2001), $\tilde{T}(Q)$ and $\tilde{\lambda}$ have the same shape for the membership function; that is, the total cost $\tilde{T}(Q)$ is also a fuzzy number.

It is clear that the determination of the optimal order quantity Q^*_D depends on the location of the price breakpoint, Q^s_c. As generally known, it is meaningless if Q^s_c is smaller than l or is larger than u, so in this study we discuss the cases of $Q^s_c\in[l,u]$. It is clear that the derivation of the closed-form of Q^*_D for the fuzzy single-period model