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Optimal inventory policy for the fuzzy newsboy problem with quantity discounts

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ABSTRACT

Newsboy models have wide applications in solving real-world inventory problems. This paper analyzes the optimal inventory policy for the single-order newsboy problem with fuzzy demand and quantity discounts. The availability of the quantity discount causes the analysis of the associated model to be more complex, and the proposed solution is based on the ranking of fuzzy numbers and optimization theory. By applying the Yager ranking method, the fuzzy total cost functions with different unit purchasing costs are transformed into convex, piecewise nonlinear functions. By proving certain properties of the ranking index of the fuzzy total cost, several possible cases are identified for investigation. After analyzing the relative positions between the price break and the minimums of these nonlinear functions, the optimal inventory policies are provided and closed-form solutions to the optimal order quantities are derived. Several cases of a numerical example are solved to demonstrate the validity of the proposed analysis method. The advantage of using the proposed approach is also demonstrated by comparing it to the classic stochastic approach. It is clear that the proposed methodology is applicable to other cases with different types of quantity discounts and more complicated cases.

1. Introduction

In today's highly competitive business environment, inventory management's ability to plan and control inventories to meet the competitive priorities is becoming increasingly important in many types of organizations [21,34]. One type of inventory problem frequently encountered with seasonal or customized products is the newsboy problem, also called the newsvendor problem or single-period stochastic inventory problem because only a single procurement is made [12,27]. Typical practical examples are the dilemmas of making a one-period decision on the quantity of newspapers that a newsboy should buy on a given day or the quantity of seasonal goods that a retailer should purchase for the current year, goods that cannot be sold the next year because of style changes.

One of the principal factors in order quantity decisions in the inventory problem is the nature of the demand. In actual applications, demand is uncertain and must be predicted. Notably, there are cases in which the probability distribution of the demand for new products is typically unknown because of a lack of historical information, and the use of linguistic expressions by experts for demand forecasting is often employed. Consequently, the decision maker faces a fuzzy environment rather than a stochastic one in these cases.

Fuzzy set theory has been applied to inventory problems with demand uncertainties attributed to fuzziness rather than randomness (for example [3–5,7–10,24–26,28,30,31,37,38,41,42]). In particular, some scholars have approached the

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newsboy problem from the standpoint of fuzzy environments. For example, Petrovic et al. [29] proposed a model for the newsboy problem where the demand is described by a discrete fuzzy set and the cost is represented by a triangular fuzzy number. Ishii and Konno [16] used the fuzziness concept to consider the shortage cost in the classic newsboy problem, although the demand was stochastic. Kao and Hsu [19] proposed a model to find the optimal order quantity of the classic newsboy problem with fuzzy demand. Li et al. [25] proposed two models for newsboy problems that have both random and fuzzy uncertainties. Recent work on fuzzy, random newsboy problems includes the studies by Xu and Hu [37] and Hu et al. [13].

There are several other studies on fuzzy newsboy problems. For example, based on fuzzy clustering and fuzzy rules, Cardoso and Gomide [2] proposed a predictive data mining model to predict newspaper demand. In supply chain environments, Xu and Zhai [39] investigated the fuzzy newsboy problem with one manufacturer and one retailer. Hu et al. [13] investigated the fuzzy random newsboy problem with imperfect quality. Dutta and Chakraborty [8] considered the product substitution policy and proposed a fuzzy single-period inventory model with fuzzy demand.

These models can be applied to find the optimal order quantity for the classic newsboy problem in fuzzy environments. However, there are many practical situations that are worth further investigation. In particular, quantity discounting (see, for example [43]), which is a price reduction for large orders offered to customers to induce them to buy in larger quantities, is a situation often encountered in practice. However, relatively few papers have been published on the fuzzy inventory problem with quantity discounts. Lam and Wong [22] applied fuzzy mathematical programming to solve economic lot-size problems with multiple price breaks, although they did not focus on newsboy problems. Ji and Shao [17] proposed a hybrid intelligent algorithm based on genetic algorithms and fuzzy simulation to solve the bi-level newsboy problem with fuzzy demand and discounts, but they did not provide analytical solutions.

The purpose of this study is to find the optimal order quantity for the single-order newsboy problem when the demand is fuzzy and quantity discounts are available. The proposed solution is based on the ranking of fuzzy numbers and optimization theory. This idea is inspired by the concept of Kao and Hsu [19], but the analysis and results are significantly different and more complex because of the nature of quantity discounts.

In the following sections, the classic newsboy problem is reviewed and the fuzzy newsboy model with quantity discounts is briefly introduced. Next, the fuzzy total cost functions with different unit purchasing costs are transformed into convex, piecewise nonlinear functions by applying the Yager ranking method. We propose and prove two properties of the ranking index of the fuzzy total cost to identify possible cases for investigation. Following this, after analyzing the relative positions between the price break and minimums of these nonlinear functions, the best inventory policies are provided and closed-form solutions for the optimal order quantities are derived. Several numerical examples are solved to demonstrate the validity of the proposed analysis method. Finally, the conclusions are presented.

2. Fuzzy newsboy model with quantity discounts

The classic newsboy problem (also called the newsvendor problem or Christmas tree problem), a famous stochastic inventory replenishment problem, can be described as follows [35]. A newsboy buys a certain amount of a product at the beginning of the period, and only a single procurement is made [11,35]. Before knowing the actual demand, the newsboy wants to decide the optimal order quantity for this product such that the expected total cost is minimized. The unit purchasing price is c, and the newsboy sells the item on the market at the selling price per unit of p ($p > c$). If the order quantity, Q, exceeds the actual demand, λ, a holding cost for the leftover items will be incurred; otherwise, a shortage cost, including the opportunity cost of lost profit and the loss of customer goodwill, will be incurred. Let h be the holding cost per unit remaining at the end of the period ($h < 0$, representing the salvage value per unit), where the salvage value is typically less than the unit cost, i.e., $c > -h$ [19]. Therefore, the incurred total cost is

$$T(Q) = cQ + p\max\{0, \lambda - Q\} + h\max\{0, Q - \lambda\}. \tag{1}$$

In practice, the main difficulty faced by the newsboy is to predict the actual demand because this typically involves uncertain factors. In classic newsboy models, the demand is described by a random variable λ with a known statistical distribution. Thus, the newsboy problem is to find the optimal order quantity, Q^*, such that the expected value of the total incurred cost is minimized. If λ follows a continuous distribution with a probability density function (pdf) of $f_\lambda(D)$, the associated optimization model is

$$\min_{Q>0} E[T(Q)] = cQ + p\int_Q^\infty (D-Q)f_\lambda(D)dD + h\int_0^Q (Q-D)f_\lambda(D)dD. \tag{2}$$

Because the function $E[T(Q)]$ has been proven to be convex in Q [12], the minimum value of $E[T(Q)]$ occurs at Q^*, the point at which the first derivative of $E[T(Q)]$ with respect to Q satisfies $dE[T(Q)]/dQ = 0$. The optimal order quantity can be obtained as

$$F_\lambda(Q^*) = \frac{p-c}{p+h}, \tag{3}$$

where $F_\lambda(\cdot)$ is the cumulative distribution function (cdf) of the random variable λ. However, if λ follows a discrete distribution with a probability mass function (pmf) of $p_\lambda(D)$, the associated optimization model is