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# A comparison of simple heuristics for multi-product dynamic demand lot-sizing with limited warehouse capacity

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ABSTRACT

The paper analyzes the problem of the replenishment of multiple products to satisfy dynamic demands when the warehouse capacity or the available inventory budget is limited. In this context the timing of replenishment lot-sizes has to be staggered to account for the capacity conflict and to provide an effective space sharing in addition to the solution of the trade-off between setup and inventory holding costs. We analyze three simple heuristics. First, we review a forward algorithm that successively builds lots by extending replenishments according to a cost-based priority rule. The second heuristic solves the lot-sizing problems independently for each product in a first step and then resolves capacity violations by a smoothing mechanism. Further, this paper adapts a heuristic for single-item uncapacitated lot-sizing that successively improves an initial lot-for-lot schedule by combining replenishments according to a cost savings-based priority rule to the multi-item capacitated problem. The performance of the three simple methods is compared in an extensive numerical study and benchmarked against the solution of a mixed-integer programming approach. The results show the different ability of the approaches to simultaneously account for the individual lot-sizing problems and the lot-staggering problem across multiple products. Especially the savings approach appears to provide better results for a broad range of problems, especially for large, tightly capacitated problems with high demand variability.



## 1. Introduction

Single-product dynamic lot-sizing referring back to the seminal paper by Wagner and Whitin (1958) and diverse lot-sizing heuristics are included in today's operations management textbooks and material requirements planning software systems. The interdependencies between different items are mostly considered in dependent demand systems like MRP and DRP systems where the (vertical, multi-stage) interaction results from the bill of material structure. In this paper, we focus on horizontal single-stage interaction between multiple products. In this context, three main aspects of multi-product lot-sizing coordination are extensively discussed in the literature, (i) joint replenishment problems (JRPs), (ii) capacitated lot-sizing and scheduling problems, and (iii) warehouse scheduling problems. The focus of this paper will be on discrete time, finite horizon, and deterministic demand models which represent straightforward extension of Wagner and Whitin (1958) to the multi-product environment. Reviews of dynamic lot-sizing heuristics are, e.g., Wemmerlöv (1982) and Zoller and Robrade (1988).

In the context of procurement, the JRP analyzes the replenishment of multiple products which are connected by a joint cost structure. Besides the individual setup and inventory holding costs for each product, a major setup cost for any replenishment (regardless of the product and the number of involved products) connects the individual decisions. For a review on deterministic and stochastic,

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as well as continuous time and discrete time, and static versus dynamic models for the JRP, see Aksoy and Erenguc (1988). Boctor et al. (2004) present alternative mixed-integer linear programming formulations and give a comparison of several heuristics for the discrete time, deterministic demand case.

If the interaction between multiple products results from competition for a (capacitated) manufacturing facility, lot-sizing and scheduling problems have to be solved. Due to the challenging research issues and the practical relevance in various industries the majority of coordinated lot-sizing models and algorithms has evolved in this field, for reviews see De Bodt et al. (1984), Bahl et al. (1987), Maes and Van Wassenhove (1988), Kuik et al. (1994), Drexl and Kimms (1997), and Pochet and Wolsey (2006).

If the inventories resulting from batch replenishments compete for limited warehouse capacity or a limited budget, replenishment quantity and timing decisions for the different products have to be coordinated. For the case of shared warehouse space (in contrast to space dedicated to each product) the replenishments have to be staggered over time such that the space released by the demands for other items can be used when storing the next incoming batch for a product. This problem is called the warehouse scheduling problem. Many contributions exist for continuous time, constant demand models and are straightforward extensions of the EOQ model, e.g., for the dedicated capacity case Johnson and Montgomery (1974), for shared capacity with staggered orders on a common cycle, Page and Paul (1976), and for more general policies Gallego et al. (1996) and Hariga and Jackson (1996). Extensions to stochastic demand are suggested by Minner and Silver (2005, 2007).

Only a few contributions deal with the deterministic, discrete time, dynamic demand problem under limited warehouse capacity. Love (1973) analyzes the single-item problem with a concave cost structure and proposes a dynamic programming algorithm with $O(T^3)$ complexity, $T$ denoting the number of (discrete) time periods. Gutiérrez et al. (2003) present an improved algorithm and discuss special cases that allow for algorithms with $O(T)$ complexity. The optimality properties derived for concave costs in the single-item case generalize to the multi-item problem as shown by Richter (1975) and Dixon and Poh (1990). For the special case of linear costs as considered in most practical applications, this property reduces to the well-known zero-inventory property as in the Wagner–Whitin model where a product is only replenished if the inventory level at the beginning of a period equals zero and if units are replenished, the corresponding batch includes the demands of consecutive periods.

Günther (1990, 1991) uses this property to successively extend the time coverage of an item's procurement lot in a forward algorithm based on a cost priority criterion to select between the products. Within a linear programming-based relaxation algorithm, Dixon and Poh (1990) suggest a smoothing heuristic that first determines independent lot-sizes for each product and in a second step removes infeasibilities by shifting replenishments. These methods are presented in more detail in Section 3. Both methods have in common that they proceed in a forward manner. This paper proposes a different method that is based on the savings idea known from vehicle routing and applied to the single-item, uncapacitated dynamic lot-sizing problem by Axsäter (1980). Here, the largest savings from combining replenishments are implemented which implicitly regards the lot-staggering problem by a flexible rather than a forward combination of demands. Karni (1981) uses the same idea and suggests an extension to single-item problems with limited production capacities.

In Section 2 we present a new mixed-integer linear programming formulation for the warehouse scheduling problem. In Section 3 we review two existing heuristic approaches and suggest a new heuristic based on the savings idea in Section 4. The approaches are compared in an extensive numerical study in Section 5.

## 2. Model formulation

In this section we present a mixed-integer programming formulation that is based on a fractional representation of the decision variables. For the solution of the mixed-integer-linear program this will lead to tighter lower bounds. Let $N$ denote the number of products, $i = 1, 2, \ldots, N$ and $T$ the number of periods, $t = 1, 2, \ldots, T$. The problem is modelled as a deterministic discrete time, finite horizon planning problem with given demands $d_{it}$ for product $i$ in period $t$. Backorders are not permitted. Each replenishment for a product at the beginning of a period is associated with a setup cost $S_i$. We assume that there exist no interdependencies between costs for the individual products as in the JRP. Replenishment lead times are assumed to be negligible. Inventories at the end of a period are subject to linear inventory holding cost $h_i$ per unit and unit of time. Without loss of generality we assume zero initial and final inventories. The warehouse capacity $W$ is assumed to be constant over the planning horizon. A unit of each product $i$ requires $a_i$ warehouse capacity units.

There exist different assumptions concerning the instant of time when the warehouse capacity constraint has to hold. Let $\alpha_i$ denote the capacity requirement of current period demand. If $\alpha_i = 1$, the inventory capacity constraint has to hold at the beginning of each period whereas if $\alpha_i = 0$, it is assumed that the current demand in period $t$ does not affect the warehouse capacity. An alternative interpretation for $\alpha_i = 0$ or 1 is that inventory depletion for item $i$ occurs either at the beginning or at the end of the period.

Let $u_{i\tau}$ denote the binary indicator variable whether an order is placed for item $i$ in period $\tau$ or not. The decision variables $x_{i\tau t}$ denote the fraction of demand for item $i$ in period $t$ that is replenished in period $\tau$. Let $h_{i\tau t}$ denote the holding cost if the entire demand for item $i$ in period $t$ is replenished in period $\tau$, i.e.

$$h_{i\tau t} = h_i(t-\tau)d_{it}, \quad i = 1, 2, \ldots, N; \ \tau = 1, 2, \ldots, T; \ t = \tau, \tau+1, \ldots, T. \qquad (1)$$