Computers and Mathematics with Applications 61 (2011) 2388–2400

Contents lists available at ScienceDirect

Computers and Mathematics with Applications

journal homepage: www.elsevier.com/locate/camwa

A volume flexible economic production lot-sizing problem with imperfect quality and random machine failure in fuzzy-stochastic environment

Debasis Das [a], Arindam Roy [b], Samarjit Kar [a,*]

[a] Department of Mathematics, National Institute of Technology, Durgapur, W.B, Pin-713209, India

[b] Department of Computer Science, Prabhat Kumar College, Contai, Purba Medinipur, W.B, Pin-721401, India

ARTICLE INFO

Article history:
Received 20 October 2009
Received in revised form 10 February 2011
Accepted 10 February 2011

Keywords:
Production lot-sizing
Imperfect quality
Random machine failure
Maintenance time

ABSTRACT

An "economic production lot size" (EPLS) model for an item with imperfect quality is developed by considering random machine failure. Breakdown of the manufacturing machines is taken into account by considering its failure rate to be random (continuous). The production rate is treated as a decision variable. It is assumed that some defective units are produced during the production process. Machine breakdown resulting in idle time of the respective machine which leads to additional cost for loss of manpower is taken into account. It is assumed that the production of the imperfect quality units is a random variable and all these units are treated as scrap items that are completely wasted. The models have been formulated as profit maximization problems in stochastic and fuzzy-stochastic environments by considering some inventory parameters as imprecise in nature. In a fuzzy-stochastic environment, using interval arithmetic technique, the interval objective function has been transformed into an equivalent deterministic multi-objective problem. Finally, multi-objective problem is solved by Global Criteria Method (GCM). Stochastic and fuzzy-stochastic problems and their significant features are illustrated by numerical examples. Using the result of the stochastic model, sensitivity of the nearer optimal solution due to changes of some key parameters are analysed.

1. Introduction

During the past few decades, attempt has been made to make inventory control models more realistic with real life industrial problems. In most of the classical economic production quantity (EPQ) model, it is assumed that items produced are of perfect quality and quality control of the product generally is not considered. However, in a production system, it is quite natural that a machine cannot produce all items perfect during a whole production period.

Salameh and Jaber [1] extended the classical economic order quantity EOQ model by considering imperfect quality items while using EOQ formulae. Later, Cardenas-Barron [2]corrected a mistake in the final formula of Salameh and Jaber's model. Goyal and Cardenas-Barron [3] then reconsidered the work of Salameh and Jaber and presented a practical approach for determining the optimal lot size. Hayek and Salameh [4] derived an optimal operating policy for the finite production model under the effect of reworking of imperfect quality items and assuming that all the defective items are repairable. Chiu [5] examined an EPQ model with scrap items and the reworking of repairable items. Wee et al. [6] developed an optimal inventory model for items with imperfect quality and shortages. Chung et al. [7] proposed a new inventory model with two

* Corresponding author. Tel.: +91 9434453186.
E-mail addresses: debasis_opt@yahoo.co.in (D. Das), royarindamroy@yahoo.com (A. Roy), kar_s_k@yahoo.com (S. Kar).

0898-1221/$ – see front matter © 2011 Elsevier Ltd. All rights reserved.
doi:10.1016/j.camwa.2011.02.015

warehouses and imperfect quality. Jaber et al. [8] presented the concept of entropy cost to extended a new model under the assumptions of perfect and imperfect quality. Jaber et al. [9] extended Salameh and Jaber [1] and assumed the percentage of defective per lot reduces according to a learning curve.

Nowadays, with the advent of multinationals, there is a stiff competition in the market and the management implements flexible manufacturing systems (FMS) for improving production efficiency. Volume flexibility that is capable of adjusting the production rate with variability of demand in the market is one of the important components in FMS. Volume flexibility helps to reduce production rate to avoid rapid accumulation of inventories. Obviously, the machine production rate is a decision variable in the case of a FMS and then the unit production cost becomes a function of production rate. Khouja and Mehrez [10] and Khouja [11] extended the EPLS model to an imperfect production process with a flexible production rate. Moon et al. [12], Gallego [13], etc. extended the EPLS model with flexible production rate by considering constant demand. Sana et al. [14] developed an economic manufacturing model in an imperfect production system where the defective items are sold in a reduced price.

When some inventory parameters are fuzzy in nature, its objective function also becomes fuzzy. After the introduction of fuzzy set theory in 1965 by Zadeh, extensive research work has been done on defuzzification of fuzzy numbers. Among these techniques Centroid Method [15], Weighted Average Method [16], Graded Mean Value Method [17], Nearest Interval Approximation Method [18], Graded Mean Integration Value [19], etc., have drawn more attention. All these techniques replace the fuzzy parameters by their nearest crisp number/interval and the reduced crisp objective function is optimized.

In reality, a machine cannot work smoothly forever because its spare parts will breakdown sooner or later. It may become out of order during its working time and there is a mean time between its failures/breakdowns. During a breakdown period, demand in the system persists, but there is no production. When inventory level becomes less than the demand, the management unit is rendered fully idle. In this paper we consider the idle time of the machine which leads to an additional cost for the loss of man-hours. We take the time between successive breakdowns of the machines to be random and the maintenance time is also considered to be random. Here, models have been formulated as profit maximization problems in stochastic and fuzzy-stochastic environments. In a fuzzy-stochastic environment the model is transferred into multi-objective problem and solved by Global Criteria Method (GCM). In order to illustrate the solution method numerical examples are provided. Sensitivity of the decision variable and total expected profit is examined to check how far the output of the model is affected by changes or errors in its input parameters.

2. Preliminaries

This section provides an introduction to fuzzy number and interval arithmetic.

Definition 1 (*Fuzzy Number*)**.** A fuzzy subset $\widetilde{A}$ of real number R with membership function $\mu_{\widetilde{A}} : R \to [0, 1]$ is called a fuzzy number if

(a) $\widetilde{A}$ is normal, i.e. there exist an element x_0 such that $\mu_{\widetilde{A}}(x_0) = 1$;
(b) $\widetilde{A}$ is convex, i.e. $\mu_{\widetilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq \mu_{\widetilde{A}}(x_1) \bigwedge \mu_{\widetilde{A}}(x_2)$ for all $x_1, x_2 \in R$ and $\lambda \in [0, 1]$;
(c) $\mu_{\widetilde{A}}$ is upper semi-continuous; and
(d) $\text{supp}(\widetilde{A})$ is bounded, here $\text{supp}(\widetilde{A}) = \{x \in R : \mu_{\widetilde{A}}(x) > 0\}$.

Example 1 (*Triangular Fuzzy Number*)**.** Triangular fuzzy number (TFN) $\widetilde{A}$ is an example of fuzzy number with the membership function $\mu_{\widetilde{A}}(x)$, a continuous mapping $\mu_{\widetilde{A}} : R \to [0, 1]$ and is defined by

$$\mu_{\widetilde{A}}(x) = \begin{cases} 0 & \text{for } x < a_1 \\ \dfrac{x-a_1}{a_2-a_1} & \text{for } a_1 \leq x < a_2 \\ \dfrac{a_3-x}{a_3-a_2} & \text{for } a_2 \leq x < a_3 \\ 0 & \text{for } x \geq a_3. \end{cases}$$

Generally, $\widetilde{A}$ is denoted by (a_1, a_2, a_3), where $a_1, a_2, a_3 \in R$.

2.1. α-cut of fuzzy number

The α-cut of a fuzzy number is a crisp set which is defined as $[\widetilde{A}]_\alpha = \{x \in R : \mu_{\widetilde{A}}(x) \geq \alpha\}$. According to the definition of fuzzy number it is seen that α-cut is a non-empty bounded closed interval, it can be denoted by

$$[\widetilde{A}]_\alpha = [A_L(\alpha), A_R(\alpha)].$$

$A_L(\alpha)$ and $A_R(\alpha)$ are the lower and upper bounds of the closed interval, where

$$A_L(\alpha) = \inf\{x \in R : \mu_{\widetilde{A}}(x) \geq \alpha\},$$

and

$$A_R(\alpha) = \sup\{x \in R : \mu_{\widetilde{A}}(x) \geq \alpha\}.$$