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Modeling and analysis of single item multi-period procurement lot-sizing problem considering rejections and late deliveries

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ABSTRACT

Integer linear programming approach has been used to solve a multi-period procurement lot-sizing problem for a single product that is procured from a single supplier considering rejections and late deliveries under all-unit quantity discount environment. The intent of proposed model is two fold. First, we aim to establish tradeoffs among cost objectives and determine appropriate lot-size and its timing to minimize total cost over the decision horizon considering quantity discount, economies of scale in transactions and inventory management. Second, the optimization model has been used to analyze the effect of variations in problem parameters such as rejection rate, demand, storage capacity and inventory holding cost for a multi-period procurement lot-sizing problem. This analysis helps the decision maker to figure out opportunities to significantly reduce cost. An illustration is included to demonstrate the effectiveness of the proposed model. The proposed approach provides flexibility to decision maker in multi-period procurement lot-sizing decisions through tradeoff curves and sensitivity analysis.

1. Introduction

Multi-period procurement lot-sizing decision seeks best tradeoffs among multiple cost objectives to determine appropriate lot-size and its timing to minimize total cost over the decision horizon. The multiple cost objectives are purchasing cost, transaction (ordering and transportation) cost, inventory holding cost and/or shortage cost. Supplier offers discounts, which tend to encourage buyer to procure larger quantities to obtain operating advantages such as economies of scale and reducing the cost of ordering and transportation. In such a scenario, product could be carried forward to a future period, incurring inventory holding cost. This means that in each period either procurement takes place or buyer has inventory carried forward from the preceding period. Smaller lot-size procurement strategy reduces inventory holding cost but increases purchasing cost and transaction cost. Procurement of larger lot-size reduces purchasing cost and transactions cost but leads to higher inventory cost. Supply chain risks such as rejections and late deliveries also affect the procurement lot-sizing decisions. Therefore, decision maker considers tradeoffs among purchasing cost, transaction cost, inventory holding cost and/or shortage cost in multi-period procurement lot-sizing decisions to minimize total cost over decision horizon.

Material Requirement Planning (MRP) involves procurement lot-sizing decisions to be made when demand is both stable as well as lumpy and the approach is spread over a finite time horizon. In the restricted case, when demand is stable and known over the decision horizon, the simple static EOQ model can find the optimum solution. Both methods fail to consider realistic constraints regarding supplier capacity, rejections, late deliveries and time dependent variations in problem parameters. The exact solution in more general situations has been obtainable by Dynamic Programming (DP). Wagner and Whitin (1958) presented a dynamic programming solution algorithm for single product, multi-period inventory lot-sizing problem. Even though DP algorithms (Aggarwal & Park, 1993; Federgruen & Tzur, 1991; Heady & Zhu, 1994; Silver & Meal, 1973) provide an optimal solution, these are considered difficult to understand and require high computational resources.

To our knowledge, there is no multi-period linear programming model available in the literature for procurement lot-sizing problem which can substitute EOQ model and DP model to overcome their limitations, and also considers price breaks and realistic constraints as well as supports Material Requirement Planning.

This paper applied an integer linear programming approach to solve multi-period procurement lot-sizing problem for single product and single supplier considering rejections and late delivery performance under all-unit quantity discount environment.

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The purpose of this paper is to:

- Develop a mathematical model to establish tradeoffs among cost objectives and determine appropriate lot-size to procure and its period to minimize total cost over the decision horizon.
- Investigate the effect of variation in problem parameters such as rejection rate, demand, storage capacity and inventory holding cost on total cost.

The paper is further organized as follows. Section 2 presents a brief literature review of the existing quantitative approaches related to procurement lot-sizing problem. In Section 3, an integer linear programming formulation is developed for multi period procurement lot-sizing problem considering all-unit quantity discounts. Section 4 presents an illustration with solution to demonstrate the effectiveness of the proposed approach. Finally, conclusions are provided in Section 5.

2. Literature review

Brahimi, Dauzere-Peres, Najid, and Nordli (2006) presented a survey of the single item lot-sizing problem for its uncapacitated and capacitated versions. Karimi, Fatemi Ghomi, and Wilson (2003) discussed a number of important characteristics of lot-sizing models, including the planning horizon, number of levels, number of products, capacity or resource constraints, deterioration of items, demand, setup structure and shortage. Ben-Daya, Darwish, and Ertogral (2008) and Robinson, Narayanan, and Sahin (2009) proposed different models and classifications of the lot-sizing problem.

Smith, Robles, and Cárdenas-Barrón (2009) formulated and solved a single item joint pricing and production decision problem over a multi-period time horizon. The objective is to maximize profits considering capacity and inventory constraints. They consider decision variables, such as sales price, production quantity, and sales amount for a single item. Buffa and Jackson (1983) proposed a goal programming model considering price, quality and delivery goals to schedule purchase for single product over a defined planning horizon.

Pratsini (2000) proposed the lot-sizing model with setup learning for the single level, multi-item, capacity constrained case. He developed a heuristic to analyze the effects of setup learning on a production schedule. The study revealed that setup learning can have unexpected results on a product depending on the relative value of its setup to holding cost ratio compared with the ratios of the other products. Benton (1991) developed a non-linear model and a heuristic solution approach for supplier selection and lot sizing under conditions of multiple items, multiple suppliers, resource limitations and all-unit quantity discounts. The objective is to minimize the total cost (purchasing, inventory and ordering costs) subject to an inventory investment constraint and shortage related constraints. In their article, Raza and Akgunduz (2008) presented a comparative study of heuristic algorithms on economic lot scheduling problem (ELSP). They showed that Simulated Annealing algorithm finds the best solution to these ELSP problems, and outperforms other meta-heuristic technique such as Dobson's heuristic, hybrid GA, Neighborhood Search heuristics and Tabu Search.

Polatoglu and Sahin (2000) suggested a multi-period purchasing policy where demand in each period is considered as a random variable, the probability distribution of which may depend on price and period. Chaudhry, Forst, and Zydiak (1993) proposed a mathematical formulation to minimize the purchasing cost for individual item over a single period considering capacity constraints, delivery performance and quality with quantity discounts. Bender, Brown, Isaac, and Shapiro (1985) described a procurement problem faced by IBM to minimize the sum of purchasing, transportation and inventory cost over the planning horizon for multiple products, multiple time periods and quantity discounts.

Ustun and Demirtas (2008) proposed an integration of ANP and achievement scalarizing function to choose the best suppliers and to find the optimal order quantities and inventory levels. Liao and Kuhn (2004) presented a multi-objective optimization model for single item assuming that all suppliers' lots simultaneously arrive at the beginning of each replenishment period. The objectives are the minimization of total cost, the total quality rejections and total late deliveries subject to capacity and demand constraints. Demirtas and Ustun (2009) developed an integrated ANP and GP approach to solve multi-period inventory lot-sizing scenario, for single product and multiple suppliers. A multi objective mixed integer linear programming model is proposed to achieve four goals: budget, aggregate quality, total value of purchasing and demand over the planning horizon. Rezaei and Davoodi (2011) proposed two multi objective mixed integer non-linear models for multi period lot-sizing problems involving multiple products and multiple suppliers. Each model is constructed on three objective functions: cost, quality and delivery. In first model shortages are not allowed while second model considered that demand during the stock-out period is back ordered.

3. Model formulation

The proposed model deals with procurement lot-sizing problem in which there is a time-varying demand for a single product over multi-periods.

3.1. Model parameters and decision variables

Following parameters and decision variables are to be adopted for mathematical formulation.

Parameters:

d_t	buyer's demand of the product in period t
p_{mt}	cost of procuring one unit of product between price break level m and $m+1$ in period t
b_m	quantity at which all-unit price breaks occur
o_t	cost of ordering in period t
t_{mt}	cost of transportation of lot-size at price break level m in period t
q_{mt}	percentage of rejected products delivered by supplier at price break level m in period t
l_{mt}	percentage of products late delivered by supplier at price break level m in period t
C_t	supplier capacity in period t
h_t	unit inventory holding cost of the product in period t
w	buyer's storage capacity
I_t	intermediate variable indicates inventory of the product, carried over from period t to period $t+1$

Decision variables:

x_{mt}	lot-size (number of units of product) that buyer procures from supplier at price break level m in period t
y_{mt}	binary variable used in separating price levels m for product in a transaction between buyer and supplier. This also separates transportation cost per procured lot-size between price break level m and $m+1$ in period t
z_t	binary variable indicating whether supplier is ordered or not in period t