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Optimal pricing, lot-sizing and marketing planning in a capacitated and imperfect production system ☆

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ABSTRACT

In the classical economic production quantity (EPQ) problem demand is considered to be known in advance. However, in the real-world, demand of a product is a function of factors such as product's price, its quality, and marketing expenditures for promoting the product. Quality level of the product and specifications of the adopted manufacturing process also affect the unit product's cost. Therefore, in this paper we consider a profit maximizing firm who wants to jointly determine the optimal lot-sizing, pricing, and marketing decisions along with manufacturing requirements in terms of flexibility and reliability of the process. Geometric programming (GP) technique is proposed to address the resulting nonlinear optimization problem. Using recent advances in optimization techniques we are able to optimally solve the developed, highly nonlinear, mathematical model. Finally, using numerical examples, we illustrate the solution approach and analyze the solution under different conditions.

1. Introduction

Traditional economic production/order quantity (EPQ/EOQ) models make various simplifying assumptions in order to arrive at a closed form solution for the optimal lot size in a production facility. For example, it is assumed that factors such as unit production cost, product's demand, and set-up cost are fixed and known in advance, and that the items produced are all of a perfect quality (Silver, Pyke, & Peterson, 1998). However, in reality, these assumptions are rarely satisfied. Therefore, to take into account the real-life situations, and to respond more appropriately to increasing competitiveness in the global business environment, recently many researchers have studied the production/inventory lot sizing models under more realistic conditions.

Salameh and Jaber (2000) extended the traditional EPQ/EOQ model considering that produced or received items are not of perfect quality. In their model, each lot contains a random fraction of imperfect quality items, where these poor-quality items can later be sold as a single batch at the end of the screening process. Maddah and Jaber (2008) resolved a flaw in the Salameh and Jaber's work and obtained a simple expression for the optimal order quantity. Moreover, they let several batches of poor-quality items to be consolidated and shipped. Jaber (2006) also extended the classical EOQ model by considering an imperfect production process in which set-up cost is not constant and, due to the learning effect, decreases after each set-up. Khan, Jaber, and Bonney (2011a) investigated the effect of both imperfect quality items and inspection errors in the EOQ models. For a comprehensive review on the extensions of a modified EOQ model for imperfect quality items the reader is referred to Khan, Jaber, Guiffrida, and Zolfaghari (2011b).

The assumption of fixed unit cost in EOQ/EPQ models has also been tackled by many researchers. For example, Cheng (1991) formulated a problem where production cost is affected by both the product's demand and the process reliability. Lee (1994), by formulating the unit cost as a function of the order quantity, took the economies of scale into account. Also, Cheng (1989b), and Jung and Klein (2001) considered the unit product's cost as a function of demand. Revising the fixed demand rate assumption has been another concern in the lot sizing literature. Among others, Lee (1993, 1994), Lee, Kim, and Victor Cabot (1996) exerted that the demand is not fixed and known in advance, rather depends on the unit product's cost, while Lee and Kim (1993) and Sadjadi et al. (2005) expressed the demand as a nonlinear function of product's price and marketing investments.

Due to the relationships between different parameters and decision variables affecting the lot sizing decisions, many mathematical models that have been recently developed in the EOQ/EPQ domain tend to be nonlinear. Kochenberger (1971) was the first to tackle the nonlinear economic order quantity problem through geometric programming (GP) method (Duffin, Peterson, & Zener, 1967). Later, Cheng (1989a) formulated an EPQ model

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Nomenclature

D	total market demand covered by manufacturer
c	unit production cost
i	percent inventory holding cost (per unit per unit time)
P	total market demand
ρ	percent manufacturer's share of total market demand
w	space requirement for each item
W	total space available for holding produced items
B	total budget available to the marketing department
a	resource requirements for each item (e.g. machine hour)
R	total resources available
$N(r)$	maintenance costs per production cycle
$Y(S, r)$	total cost of interest and depreciation for the production process in each cycle

Decision variables

p	unit price of the product,
M_i	volume of investments in marketing method $i = 1,\ldots,I$, per unit time
Q	economic production quantity (EPQ)
q	quality level of the product from the customers point of view (i.e., percent of satisfied customers' with the product)
r	reliability level of the production process (percent of non-defective items in a batch)
S	set-up cost (representing process flexibility)

with flexibility and reliability considerations via GP, and Cheng (1989b) applied GP for solving lot sizing problems where the cost per unit is a function of demand. Lee (1993) proposed GP formulations for an EOQ problem where lot-size and selling price were decision variables, and cost was assumed to be a function of the lot-size. Kim and Lee (1998) considered a fixed and variable capacity problem for joint determination of price and lot-size, where demand was assumed to be price-dependent. Chen (2000) proposed a profit-maximizing inventory model for joint optimization of quality level, selling quantity, and purchasing price of a product for intermediate firms. He assumed that the selling price, supply rate of the product, and the fixed selling costs were power functions of decision variables, and solved the resulting model through geometric programming method. Jung and Klein (2006) analyzed different EOQ models under profit maximization criterion where disparate nonlinear functions for cost and demand were proposed. GP technique and derivative based classical first order conditions were used to derive optimal solutions.

Another issue in the lot sizing area that has drawn the attention of many researchers is integrated production and marketing planning. In one of the early works in this domain, Lee and Kim (1993) incorporated pricing and marketing decisions in the context of the traditional lot-sizing problem. They considered a profit maximizing problem in which demand was perceived as a function of product price and marketing expenditures. They proposed a solution method for their problem and compared it with some heuristic algorithms in order to show effectiveness of their approach. Sadjadi, Oroujee, and Aryanezhad (2005) formulated a profit maximizing GP model for optimal production and marketing planning where demand depends on price and marketing expenditures, and production cost is inversely related to the lot size. Additionally, they considered a flexible production rate and derived a closed-form solution for the problem. Fathian, Sadjadi, and Sajadi (2009) investigated a pricing model for electronic products. They developed a mathematical model assuming that demand depends on product's price, marketing and service expenditures, and unit production cost is a function of demand. They used the GP dual method to find the global solution for the problem. Esmaeili and Zeephongsekul (2010) proposed several seller–buyer supply chain lot sizing models where market demand is a nonlinear function of price and marketing expenditures and proposed non-cooperative stackelberg games for solving the developed problems.

Consideration of flexibility and/or reliability of production process have been another issue dealt with by some researchers in the geometric programming-based EOQ/EPQ models. Leung (2007) proposed an EPQ model with a flexible and imperfect production process in which interest and depreciation costs depend on the process flexibility and its reliability. He formulated the proposed problem using geometric programming method and obtained a closed-form optimal solution for it. Bag, Chakraborty, and Roy (2009) considered a production inventory model in an imprecise and uncertain mixed environment. In their work, demand is modeled as a fuzzy random variable and the set-up cost, reliability of the production process, and production period are considered as decision variables. The unconstrained signomial GP method is used to determine the optimal decision. Sadjadi, Aryanezhad, and Jabbarzadeh (2010a) addressed a production-marketing problem in the context of unreliable production process. They jointly optimized the lot size, marketing expenditures, set-up cost, and reliability of the production process through a nonlinear posynomial geometric program and obtained a closed form solution. Fathollah Bayati, Rasti Barzoki, and Hejazi (2011) established a rather thorough integrated EPQ model where various types of non-perfect products – perfect, imperfect, defective but reworkable, and non-reworkable defective – are taken into consideration. They simultaneously determined lot size, marketing expenditure, selling price, set-up cost, and inventory holding cost using the GP method.

In this paper, we investigate an integrated pricing, lot-sizing, and marketing planning model in which optimal levels for product's quality along with flexibility and reliability of the production process also need to be determined. This paper makes several contributions to the current literature. First of all, we have comprehensively included all the relevant and meaningful factors and decision variables that different researchers considered in formulating demand, unit production cost, and interest and depreciation cost functions in our proposed power functions. Secondly, to the best of our knowledge, for the first time we have accounted for the relationship between process reliability and maintenance costs in the area of integrated lot-sizing and marketing planning. Third, in order to better model the situation as it is in the real world, we have incorporated constraints on production capacity, storage space and marketing budget in our mathematical model. And finally, unlike other works in this domain which find optimal solutions for geometric programs with zero or one degrees of difficulty, by using recent advances in optimization software, we have been able to determine optimal levels for decision variables in a geometric program with a high degree of difficulty.

The remainder of the paper is organized as follows. In the next Section, we present the problem statement and underlying assumptions for developing the profit maximization GP model for the problem under consideration. Mathematical formulation of the GP model is presented in Section 3. Solution approach, followed by a numerical example for illustrative purposes, is the subject of Section 4. Sensitivity analysis of the optimal solution is