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Trade-off between accuracy and interpretability: Experience-oriented fuzzy modeling via reduced-set vectors ☆

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ABSTRACT

This paper focuses on accuracy and interpretability issue of fuzzy model approaches. In order to balance the trade-off between both of the aspects, a new fuzzy model based on experience-oriented learning algorithm is proposed. Firstly, support vector regression (SVR) with presented Mercer kernels is employed to generate the initial fuzzy model and the available experience on the training data. Secondly, a bottom-up simplification algorithm is introduced to generate reduced-set vectors for simplifying the structure of the initial fuzzy model, at the same time the parameters of the simplified model derived are adjusted by a hybrid learning algorithm including linear ridge regression algorithm and gradient descent method based on a new performance measure. Finally, taking the results from two-dimensional sinc function approximation and fuzzy control of the bar and beam system, the proposed fuzzy model preserves nice accuracy and interpretability.

1. Introduction

Fuzzy systems have demonstrated their ability for modeling or control in a huge number of applications. The key to their success and interest is the ability to incorporate human knowledge, so that the information mostly provided for many real-world systems could be discovered or described by fuzzy statements. Developing and establishing fuzzy systems is *fuzzy modeling* (FM), which considers model structures in the form of fuzzy rule-based systems and constructs them by means of different parametric system identification techniques [1].

In recent years, the interest in data-driven approaches to FM has increased. On the basis of limited training data set, fuzzy systems can be effectively modeled by means of some learning mechanisms, and the fuzzy model after learning tries to infer the true information. In order to assess the quality of the obtained fuzzy models, there are two contradictory requirements [2]: interpretability, capability to express the behavior of the real system, and the accuracy, capability to faithfully represent the real system. In general, the search for the desired trade-off is usually performed from two different perspectives, mainly using certain mechanisms to improve the interpretability of initial accuracy fuzzy models, or to improve the accuracy of good interpretable fuzzy models. For example, Gustafson–Kessel or Gath–Geva fuzzy clustering is usually employed to identify the initial accuracy fuzzy models [3], and subsequently similarity measures of fuzzy sets or orthogonal transformation methods are applied to improve the interpretability of the obtained model even at the expense of losing certain accuracy [4,5]; otherwise, Delgado uses fuzzy C-means clustering to generate interpretable fuzzy models, and then tunes the parameters with genetic algorithm to increase accuracy [6]. Here, the accuracy is the performance of fuzzy systems in terms of approximation evaluated over training data set, since there is always the chance of indirectly improving the accuracy in terms of generalization when improving interpretability. Actually, the accuracy requires the

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fulfillment of two important aspects: approximation and generalization. However, most studies about search of a balance of interpretability–accuracy in FM only focus on the trade-off between approximation accuracy and interpretability, and leave the generalization alone. It is just conceivable that the possibility mentioned above may happen. Hence, while improving interpretability for obtaining a transparent rule base, we lose certain approximation but gain uncertain generalization. Evaluating both approximation and generalization at the same time is still an open issue when improving interpretability. The main aim of this paper is to propose some learning mechanism in order to establish an interpretable fuzzy model which gives excellent approximation and generalization performance.

It is well known that *Support Vector Machine* (SVM) has been shown to have the ability of generalizing well to unseen data, and giving good balance between approximation and generalization [7]. Thus, some researchers have been inspired to combine SVM with FM in order to take advantages of both of approaches, human interpretability and nice performance. Therefore, support vector learning for FM has evolved into an active area of research [8–11]. Chan et al. [8] proposed a support vector neural network (SVNN) with radial basis function to the modeling of nonlinear dynamic systems. The structure of SVNN has the same form of the fuzzy basis functions expansion model, but the weights of SVNN were estimated using linear least-squares algorithm so that the derived model has lost the excellent generalization ability of original model. Chiang et al. [9] tried to construct fuzzy model with regarding fuzzy basis function whose denominator was removed as kernel function of the SVM framework, but the optimal choice of fuzzy rules depends heavily on the training data set, particularly on the learning process of the support vector modeling, and furthermore the effects of hyperparameters settings on the properties of the obtained fuzzy inference system was not yet known. The ε-insensitive learning algorithm from SVR was introduced by Leski [10] to estimate consequence parameters with control of the fuzzy model complexity, and the antecedent structure and parameters were represented as data-dependent kernel matrix obtained by fuzzy C-means clustering algorithm in input space. Clearly, the rule number of obtained fuzzy model is equivalent to the cluster number which is usually difficult to be chosen for the balance of interpretability and accuracy of fuzzy model even though there are some validation measures for fuzzy C-means. Shen et al. [11] tried to construct new support vector fuzzy adaptive network (SVFAN) whose rule number directly depends on the number of support vectors. Similarly, the performance is highly determined by the selection of hyperparameters in SVR.

In general, in the study of combining SVM with FM, nearly all the methods extract support vectors for the use of fuzzy rules generation; thereupon one support vector corresponds to one fuzzy rule. This makes the number of support vector equals to the rule number, which is usually made as few as possible since it is easier to enhance interpretability of fuzzy model as the rule number decreases. However, as we shall see in examples or experiments, optimal generalization performances of SVM are achieved with the number of support vectors more or less than 50% training samples [12]. If we tune the hyperparameters of SVM to reduce the number of support vectors, then the desirable generalization performances will loss. In other words, when we extract support vectors of the ordinary solution of SVM for generating fuzzy rules, the optimal model selection procedures, such as cross validation, leave one out, Bayesian evidence framework, etc. [13], will produce too many fuzzy rules, which will result some redundancy rules in rule base.

In this paper, we solve the trade-off problem between the interpretability and accuracy using a new fuzzy model based on experience-oriented learning algorithm. Firstly, a connection between fuzzy membership functions and Mercer kernels is established to get interpretable Mercer kernels, and then the SVR with presented kernels is employed to extract support vectors for generating initial fuzzy rules. Based on simple equivalent transform, the obtained SVR model is characterized by TS fuzzy inference procedure. Thus, the experience acquired from SVR becomes available in the form of fuzzy model, and its prudent usage could be highly advantageous. Secondly, in order to preserve nice performance while improving interpretability, a bottom-up simplification algorithm is employed to generate reduced-set vectors for simplifying the initial structure of fuzzy model, at the same time the parameters of simplified model derived are adjusted by a hybrid learning algorithm including linear ridge regression algorithm and gradient descent method. The performance index is measured by the difference between the desired output and actual output plus the experience of SVR, i.e., the difference between the actual output and output of SVR.

The rest of this paper is organized as follows: SVR and fuzzy model are briefly summarized and discussed in Section 2. Interpretable Mercer kernels are studied in Section 3. Section 4 describes the new fuzzy model proposed, and presents the associated learning algorithms. Some numerical results about the proposed fuzzy model are shown in Section 5. Finally, Section 6 concludes this paper.

2. Review of earlier works and analysis

2.1. Fuzzy models and support vector regression

Given observation data from an unknown system, data-driven methods aim to construct a decision function $f(\mathbf{x})$ that can serve as an approximation of the system. Indeed, both of fuzzy models and SVR are employed to describe the decision function.

Fuzzy models characterize the system by a collection of interpretable if-then rules, and a general Takagi–Sugeno fuzzy model which consists of a set of rules with the following structure [3]:

$$R_i : \text{If } x_1 \text{ is } A_{i1} \text{ and } x_2 \text{ is } A_{i2} \text{ and } \cdots x_d \text{ is } A_{id}, \text{ then } \quad y_i = g_i(\mathbf{x}, \beta_i) \quad \text{for } i = 1, 2 \ldots, c. \tag{1}$$