A periodic review inventory model involving fuzzy expected demand short and fuzzy backorder rate

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Abstract

The purpose of this paper is to extend [Ouyang, L. Y., Chuang, B. R. (2001). A periodic review inventory-control system with variable lead time. *International Journal of Information and Management Sciences, 12*, 1–13] periodic review inventory model with variable lead time by considering the fuzziness of expected demand shortage and backorder rate. We fuzzify the expected shortage quantity at the end of cycle and the backorder (or lost sales) rate, and then obtain the fuzzy total expected annual cost. Using the signed distance method to defuzzify, we derive the estimate of total expected annual cost in the fuzzy sense. For the proposed model, we provide a solution procedure to find the optimal review period and optimal lead time in the fuzzy sense so that the total expected annual cost in the fuzzy sense has a minimum value. Furthermore, a numerical example is provided and the results of fuzzy and crisp models are compared.
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1. Introduction

In recent years, Yao and others (Chang, Yao, & Ouyang, 2006; Chang, Yao, & Lee, 1998; Lee & Yao, 1998, 1999; Lin & Yao, 2000; Ouyang & Yao, 2002; Yao & Lee, 1996, 1999; Yao, Chang, & Su, 2000; Yao & Su, 2000; Yao & Chiang, 2003; Yao, Ouyang, & Chang, 2003) have contributed several articles by applying the fuzzy sets theory to deal with the production/inventory problems. In Lee and Yao (1999), Yao and Lee (1999) and Yao et al. (2000), they presented the fuzzy EOQ models for the inventory with backorders. Chang et al. (1998) fuzzfied the backorder quantity to be the triangular fuzzy number, while the order quantity is an ordinary variable. By contrast, Yao and Lee (1996) fuzzified the order quantity to be the triangular fuzzy number, while the backorder quantity is an ordinary variable. In Yao and Su (2000), the total demand was fuzzified to be the interval-value fuzzy set. In Lee and Yao (1998) and Lin and Yao (2000), the authors discussed the production inventory problems and proposed the fuzzy EPQ models, where Lee and Yao (1998) fuzzified the demand quantity and production quantity per day and Lin and Yao (2000) fuzzified the production quantity

per cycle, all to be the triangular fuzzy numbers, and the fuzzy total costs were derived using the extension principle.

Besides, there are several authors presented various fuzzy inventory models. For example, Petrovic and Sweeney (1994) fuzzified the demand, lead time and inventory level into triangular fuzzy numbers in an inventory control model, and then decided the order quantity by the method of fuzzy propositions. Chen and Wang (1996) fuzzified the demand, ordering cost, inventory cost, and backorder cost into trapezoid fuzzy numbers and used the functional principle to obtain the estimate of total cost in the fuzzy sense. Vujosevic, Petrovic, and Petrovic (1996) fuzzified the ordering cost and holding cost into trapezoid fuzzy numbers in the total cost of an inventory without backorder model and obtained the fuzzy total cost. Then they did the defuzzification by using centroid and gained the total cost in the fuzzy sense.

In this paper, we recast the Ouyang and Chuang's (2001) model by introducing the fuzziness of expected demand shortage and backorder rate, and then using the signed distance method to defuzzify, we derive the estimate of total expected annual cost in the fuzzy sense. The same signed distance method has also been used in Yao and Wu (2000), Yao and Chiang (2003), Lin and Yao (2003), Chiang, Yao, and Lee (2005), and others.

This article is organized as follows. In Section 2, some definitions and properties about fuzzy sets, which will be needed later, are introduced. Section 3, explores the inventory problem for a periodic review model with variable lead time. Specifically, in Section 3.1, we consider the crisp case, and given the basis of the notation and assumptions. In Sections 3.2 and 3.3, we consider the fuzzy case, so we fuzzify the expected demand shortage and backorder rate. In Section 4, we derive the optimal review period and optimal lead time by minimizing the estimate of total cost in the fuzzy sense. A numerical example is provided to illustrate the results. In Section 5, we discuss some problems for the proposed model. Finally, some concluding remarks are given in Section 6.

2. Preliminaries

In order to consider the fuzziness of an inventory problem, we need some definitions and property relative to this study. We state them in the following.

Definition 2.1. Let $\widetilde{D}$ be a fuzzy set on $R = (-\infty, \infty)$ and $0 \leqslant \alpha \leqslant 1$, the $\alpha - cut\ D(\alpha)$ of $\widetilde{D}$ consists of the points x such that $\mu_{\widetilde{D}}(x) \geqslant \alpha$, that is, $D(\alpha) = \{x | \mu_{\widetilde{D}}(x) \geqslant \alpha\}$.

Let Σ be the family of the fuzzy sets $\widetilde{D}$ on R with which the $\alpha - cut\ D(\alpha) = \{x | \mu_{\widetilde{D}}(x) \geqslant \alpha\} = [D_L(\alpha), D_U(\alpha)]$ exists for every $\alpha \in [0, 1]$, where $D_L(\alpha)$ and $D_U(\alpha)$ are continuous functions on $\alpha \in [0, 1]$, then by the Decomposition Principle (see, e.g. Kaufmann & Gupta, 1991), we have

$$\widetilde{D} = \underset{0 \leqslant \alpha \leqslant 1}{\cup} \alpha D(\alpha) = \underset{0 \leqslant \alpha \leqslant 1}{\cup} [D_L(\alpha), D_U(\alpha); \alpha]. \tag{2.1}$$

A new ranking method for fuzzy numbers, namely the signed distance has been first introduced by Yao and Wu (2000), and it has been utilized in many studies (e.g. Yao & Chiang, 2003; Chiang et al., 2005). Here we describe the concept of the signed distance on Σ which will be needed later. We first consider the signed distance on R.

Definition 2.2. For any a and $0 \in R$, define the *signed distance* of a to 0 as $d_0(a, 0) = a$.

If $a > 0$, implies that a is on the right-hand side of origin 0 with distance $d_0(a, 0) = a$; and if $a < 0$, implies that a is on the left-hand side of origin 0 with distance $-d_0(a, 0) = -a$. So, we called $d_0(a, 0) = a$ is the *signed distance* of a to 0.

For any $\widetilde{D} \in \Sigma$, from Eq. (2.1), we have

$$\widetilde{D} = \underset{0 \leqslant \alpha \leqslant 1}{\cup} [D_L(\alpha), D_U(\alpha); \alpha]. \tag{2.2}$$

And for every $\alpha \in [0, 1]$, there is an one-to-one mapping between the α-level fuzzy interval $[D_L(\alpha), D_U(\alpha); \alpha]$ and real interval $[D_L(\alpha), D_U(\alpha)]$, that is, the following correspondence is one-to-one mapping: