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Optimal price and order size under partial backordering incorporating shortage, backorder and lost sale costs

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Abstract

In this paper, we consider the pricing and lot-sizing problem for a product subject to general rate of deterioration and partial backordering. We use impatience functions to model backlogging of demand. We show that even when lost sale and backorder costs are present, the problem is well posed in the reduced space. We provide an iterative procedure for solving the overall problem. We describe structural properties of the solution for the new model and comment on the recent work incorporating backorder cost. We illustrate the solution procedure for the new model with examples.

Keywords: Order size; Pricing; Backordering; Perishable good

1. Introduction

The idea of an ''impatient'' customer in backlogging situation was proposed in Abad (1996). It was suggested that customers do not like to wait and therefore that the fraction of customers who choose to place backorders be a decreasing function of waiting time. Two specific examples of functions for backlogging of demand were given. If τ is the waiting time (i.e., time till the new supply becomes available and the backorder is filled), the fraction of customers backordering can be modeled as $B(\tau) = k_0 e^{-k\tau_1}$ or $B(\tau) = k_0/(1+k_1\tau)$, k_0, k_1 being parameters. These two functions—the exponential rate and the hyperbolic rate with respect to waiting time—have been used to model backordering in several recent studies.

San Jose et al. (2006) consider the problem of determining the lot size and the backorder level when demand is backlogged at the exponential rate. Demand is assumed to be constant and the order and lost sale costs are included. The authors coined the term ''impatience function'' to refer to the functions for modeling backlogging of demand. They have proposed a continuous two-piece function [San Jose et al. (2005a)] as well as a discontinuous step function [San Jose et al. (2005b)] for modeling backlogged demand. In the step function, backlogging rate is equal to 1 (i.e., full backordering) when waiting time is less than the specified fixed period and 0 (i.e., complete lost sale) when the waiting time is more than the fixed period. Skouri and Papachristos (2003) formulate a production lot size model where production rate, demand rate and deterioration rate are exogenous, time-varying functions and demand is backlogged with the hyperbolic rate.

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There also have been studies in which demand is considered to be price sensitive and the selling price is an endogenous variable. Abad (2001) considers the pricing and lot-sizing problem for infinite horizon. However, he ignores the shortage cost, the lost sale and the backorder cost in his analysis. Papachristos and Skouri (2003) consider the pricing and lot-sizing problem for infinite horizon when the supplier offers a lot size-based quantity discount on materials, time to deterioration is described by Weibull function and demand is backlogged at the hyperbolic rate. They include the lost sale cost and the shortage cost (i.e., cost per unit short) in their analysis. They assume that the demand rate is a convex decreasing function of selling price and that the inventory cycle time is fixed. Dye (2007) considers the pricing and lot-sizing problem for infinite planning horizon assuming the demand rate to be convex, decreasing function of selling price and the revenue to be a concave function of selling price. He assumes that the deterioration rate is time varying and that backlogging occurs at the hyperbolic rate. He includes only the lost sale cost and the cost of carrying backorders but excludes the shortage cost in his analysis. Dye et al. (2007) consider a similar problem except that backlogging occurs at the exponential rate. They again exclude the shortage cost in their analysis (see also Chang et al., 2006).

In this paper we consider the pricing and lot-sizing problem for an infinite planning horizon in a general framework. We assume that the demand rate is a decreasing function of price and that the marginal revenue is an increasing function of price. We include all three costs—the lost sale cost, the cost of carrying backorders and the shortage cost in our analysis. Furthermore, we use a general (continuous and smooth) impatience function to model the backlogging phenomenon. We provide an algorithm to determine the optimal solution. We also highlight some structural properties of the optimal solution.

2. Model formulation

The assumptions underlying the model are

1. The planning horizon is infinite.
2. The entire order quantity is received at the same time.
3. The good decays at a general rate; i.e., the decay rate is any differentiable function of time.
4. Demand is represented by a general function; i.e., the demand function can be any twice differentiable function of price subject to two conditions (described later in this section).
5. There is shortage cost, backorder cost as well as the lost sale cost.

The pattern of variation within the inventory cycle for our case is shown in Fig. 1. Let

$I(t)$	net stock (on hand-backorders) level at time t
T	the length of the duration over which net stock is positive (see Fig. 1)
Ψ	the length of the duration over which net stock is less than or equal to zero (see Fig. 1)
h	inventory carrying cost for the vendor (\$/unit/period)
$\sigma(t)$	a coefficient representing instantaneous decay rate. Assumed to be non-negative and bounded
$\sigma(t)\ I(t)$	wastage rate at time t (units/period)
K	order cost
v	unit purchase cost for the reseller
p	selling price within the inventory cycle
$D(p)$	demand rate (units/period).

In addition, let

c_1	shortage cost per unit
c_2	backorder cost per unit per unit time
c_3	lost sale cost per unit.

The following assumptions are made concerning the demand function:

(i) $D' = \dfrac{\mathrm{d}D(p)}{\mathrm{d}p} < 0 \quad \text{for all} \quad p \in (0, \infty)$

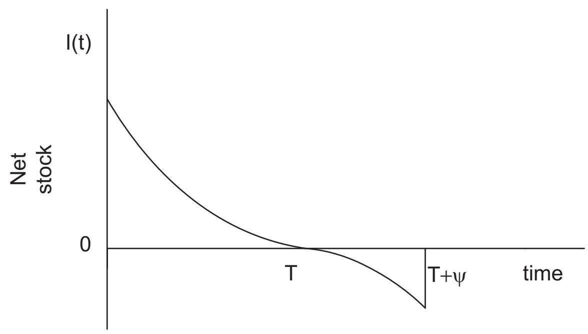

Fig. 1. The pattern of net stock.