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An integrated vendor–buyer inventory model with backorder price discount and effective investment to reduce ordering cost

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ABSTRACT

The single-vendor single-buyer integrated production inventory system has been an object of study for a long time, but little is known about the effect of investing in reducing ordering cost on the integrated inventory models with backorder price discount and variable lead time. The purpose of this article is to investigate in the continuous review model with backorder price discount and variable lead time to effectively increase investment and to reduce the joint expected annual total cost. The integrated strategy discussed here is one in which the buyer orders a quantity, then the vendor produces n times order quantity in each production cycle, in order to reduce setup cost. In addition, the buyer offers backorder price discounts to the customers that may motivate the customers' desire for backorders, and buyer ordering cost can be reduced through effective investment. An integrated inventory model is established to find the optimal solutions of order quantity, ordering cost, backorder price discount, lead time, and the number of shipments from the vendor to the buyer in one production run, so that the joint expected annual total cost incurred has the minimum value. Furthermore, numerical examples are used to demonstrate the benefits of the model.

1. Introduction

The integrated inventory management system is a common practice in the global markets and provides economic advantages for both the vendor and the buyer. In recent years, most integrated inventory management systems have focused on the integration between vendor and buyer. Once they form a strategic alliance in order to minimize their own cost or maximize their own profit, then trading parties can collaborate and share information to achieve improved benefits. Therefore, several authors (e.g., Amasaka, 2002; Ben-Daya & Hariga, 2004; Bylka, 2003; Chang, Ouyang, Wu, & Ho, 2006; Hoque & Goyal, 2006; Ouyang, Wu, & Ho, 2007a; Pan & Hsiao, 2005; Villa, 2001; Viswanathan, 1998; Yang & Wee, 2001; Zhang, Liang, Yu, & Yan, 2007) have presented the integrated inventory management system.

The integration between vendor and buyer for improving the performance of inventory system control has been discussed for years. Goyal (1976) is among the first who analyzed an integrated inventory model for a single-vendor single-buyer system. The framework he proposed has encouraged many researchers to present various types of integrated inventory system. Banerjee (1986) modified Goyal's (1976) model and presented a joint economic-lot-size model where a vendor produces for a buyer to order on a lot-for-lot basis. Goyal (1988) further generalized Banerjee's (1986) model by relaxing the assumption of the lot-for-lot policy of the vendor and suggested that the vendor's economic production quantity should be a positive integer multiple of the buyer's purchase quantity. Ha and Kim (1997) further generalized Goyal's (1988) model and presented an integrated lot-splitting model of facilitating multiple shipment in small lots. Hill (1999) proposed a more general batching and shipping policy involving the successive shipment size of the first m shipments increases by a fixed factor and remaining shipments would be equal sized. In a recently study, Pan and Yang (2002) generalized Goyal's (1988) model by considering lead time as a decision variable and obtained a lower joint total expected cost and shorter lead time. Yang and Pan (2004) considered variable lead time and quantity improvement investment with normal distributional demand in the model proposed in Pan and Yang (2002), Ouyang, Wu, and Ho (2004) extend Pan and Yang (2002) and developed a single-vendor single-buyer integrated production inventory model under the assumption that the lead time demand is stochastic and lead time is decision variable.

All the aforementioned integrated vendor–buyer inventory systems treat the ordering cost and/or lead time as constants. However, in the practical market, ordering cost and lead time can be controlled and reduced in various ways. For example, lead time can be reduced at an added crashing cost; ordering cost reduction can be attained through worker training, procedural changes, and specialized equipment acquisition; in other words, the lead time is controllable, and the ordering cost can be reduced through

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further investment. It has been a trend by shortening the lead time and reducing ordering cost; we can lower the safety stock, reduce the stockout loss, and improve the service level to the customer so as to increase the competitive edge in business.

On the other hand, in the real market, as unsatisfied demands occur, we can often observe that some customers may prefer their demands to be backordered, and some may refuse the backorder case. When a shortage occurs, many factors may affect the customers' willingness of accepting backorders. For example, for well-famed products or fashionable goods such as certain brand gum shoes, hi-fi equipment, cosmetics, and clothes, customers may prefer to wait for backorders. Hence, how to motivate the customers to wait for backorders is a valuable problem. This means that we should endeavor to generate high customer loyalty so that the customers would like to accept backorders. The factor is an offering of a price discount from the buyer to customers (see, Chuang et al., 2004; Ouyang, Chuang, & Lin, 2007b; Pan & Hsiao, 2001). In general, provided that a buyer could offer a price discount on the stockout item by negotiation to secure more backorders, it may make the customers more willing to wait for the desired items. Through controlling a price discount, we could generate high customer loyalty. This means that we could reduce cost of lost-sales and reduce holding cost, and then minimize the relevant inventory total cost. For example, Procter & Gamble, Southwest Airlines, Nike, Disney, Nordstrom, Wal-Mart, McDonald's, Marriott Hotels, and several Japanese (Sony, Toyota, Canon) and European (IKEA, Club Med, Bang & Olufsen, Electrolux, Nokia, Lego, Tesco) companies. These companies focus on the customer and are organized to respond effectively to changing customer needs. Undoubtedly, these companies endeavor to generate high customer loyalty, so that, by price discount, they can raise the customer's incentive to wait for backorder (see Kotler and Keller (2006, chap. 2)). In other words, the bigger the discount, the bigger the advantage to the customers, and hence, a larger number of backorder ratio may result. This phenomenon reveals that, as unsatisfied demands occur during the stockout period, how to find an optimal backorder ratio through controlling a price discount from a buyer to minimize the relevant inventory total cost is a decision-making problem worth discussing.

In this paper, we attempt to model an integrated vendor–buyer inventory system with controllable lead time, backorder price discount, and we further consider two widely used investment cost functional forms, logarithmic and power, to reduce ordering cost. In the literature, Porteus (1985) first introduced the concept and developed a framework for investing in reducing EOQ model set-up cost. The framework he proposed has encouraged many researchers to examine set-up/ordering cost reduction (e.g., Keller & Noori, 1988; Kim, Hayya, & Hong, 1992; Nasri, Affisco, & Paknejad, 1990; Paknejad, Nasri, & Affisco, 1995). We examine two continuous review integrated inventory models in which the buyer offers backorder price discount to the patient customers with outstanding orders during the shortage period to secure customer orders, and analyze the effects of increasing investment to reduce the ordering cost. We minimize the joint expected total annual cost per unit time by simultaneously optimizing the buyer's order quantity, ordering cost, backorder price discount, lead time, and the number of lots-the number of shipments per production run from the vendor to the buyer. Furthermore, an iterative procedure is developed to find the optimal solution. Finally, numerical examples are presented to illustrate the proposed models.

This paper is organized as follows. In the next section, the notation is presented. In Section 3, we mathematically formulate a vendor–buyer inventory model with backorder price discount and capital investment, two forms of capital investment cost function, logarithmic and power, are developed, and an efficient algorithm is developed to find the optimal solution. Two numerical examples are provided to demonstrate the results in Section 4. Section 5 we draw some concluding and give suggestions for the future research.

2. Notation

The mathematical model in this paper is developed on the basis of the following notation.

Notation:

D	average demand per year
S	vendor's set-up cost per set-up
R	production rate at the vendor
A_0	original ordering cost (before any investment is made)
A	ordering cost per order (a decision variable)
Q	order quantity (a decision variable)
h_v	vendor's holding cost per unit per year
h_b	buyer's holding cost per unit per year
n	the number of shipments per production run from the vendor to the buyer (a decision variable)
r	reorder point
$I(A)$	capital investment required to achieve ordering cost A, $0 < A \leqslant A_0$
τ	fractional annual opportunity cost of capital
β	fraction of the shortage that will be backordered at the buyer's end, $0 \leqslant \beta < 1$
β_0	upper bound of the backorder ratio, $0 \leqslant \beta < 1$
π_x	backorder price discount offered by the supplier per unit (a decision variable)
π_0	marginal profit (i.e., cost of lost demand) per unit
L	length of lead time (a decision variable)
X	lead time demand
$f_x(x)$	the probability density function (pdf) of X ith finite mean DL and standard deviation $\sigma\sqrt{L}$, where σ denotes the standard deviation of the demand per unit time
$E(\cdot)$	mathematical expectation
x^+	maximum value of x and 0, i.e., $x^+ = Max\{x, 0\}$

3. The basic model

In this paper, we restrict our attention to an integrated inventory model consisting of a single vendor who provides one type of product to a single buyer. The integrated inventory model is designed as follows. If the buyer orders quantity Q, then the vendor produces nQ at one set-up, with a finite production rate R, and $R > D$, in order to reduce its set-up cost, and then for reducing the inventory cost, the vendor as soon as delivers the lot size Q to the buyer over n times, where n is a positive integer. Therefore, the length of each production cycle for the vendor is nQ/D and the length of each ordering cycle for the buyer is Q/D.

On the other hand, we assume that the integrated production inventory model allows shortages with partial backorder. Inventory is continuously reviewed. Replenishments are made whenever the inventory level falls to the reorder point r. That is, a new order is placed whenever the reorder point is reached. At times, the order quantity of Q units will arrive before inventory reaches zero. However, at other times, higher demand will cause a stockout before a new order is received. The lead time demand follows a normal distribution, so the lead time demand X has a pdf . $f_x(x)$ with finite mean DL and standard deviation $\sigma\sqrt{L}$, and the reorder point r = expected demand during lead time + safety stock (SS), and SS = $k \times$ (standard deviation of lead time demand), i.e., $r = DL + k\sigma\sqrt{L}$ where k is the safety factor and satisfies $P(X > r) = q$, q represents the allowable stockout probability during