Lifetime consumption and investment: Retirement and constrained borrowing

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Abstract

Retirement flexibility and inability to borrow against future labor income can significantly affect optimal consumption and investment. With voluntary retirement, there exists an optimal wealth-to-wage ratio threshold for retirement and human capital correlates negatively with the stock market even when wages have zero or slightly positive market risk exposure. Consequently, investors optimally invest more in the stock market than without retirement flexibility. Both consumption and portfolio choice jump at the endogenous retirement date. The inability to borrow limits hedging and reduces the value of labor income, the wealth-to-wage ratio threshold for retirement, and the stock investment.
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1. Introduction

How does retirement affect consumption and investment? Our paper offers two new results:

- Consumption jumps at retirement because preferences are different when not working (and preferences are not additively separable over consumption and leisure).
- Investment jumps at retirement because retirement is irreversible and human capital's beta does not go to zero as the agent approaches the retirement boundary.

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The first result resolves an empirical puzzle in the literature, and the second result is an empirical prediction that is consistent with anecdotal evidence.[1] We have other results that refine or extend ideas in the literature:

- When wages are not related to market returns, agents' tendency to work longer in expensive states in which the market is down gives labor income a negative beta that makes portfolio choice even more aggressive when young than predicted by the idea that the total portfolio equals bond-like human capital plus financial capital chosen to manage overall risk exposure.
- The above result can be reversed when wages move with the market.
- When the risk in human capital is due partly to wage uncertainty, hedging of human capital is less effective when it is not possible to borrow against future labor income or when the correlation between the wage rate and the market is low, and is dampened even when the no-borrowing constraint is not currently binding.

These results are derived analytically in a consistent framework that yields rich empirical predictions and still hold even when labor income is unspanned by the financial market. We hope that these analysis and extensions will lend themselves to the study of policy questions in insurance, pensions, and retirement.

We solve three models to isolate the effects on the optimal consumption and investment strategy of retirement flexibility with and without borrowing against future labor income. We derive almost explicit solutions (at least parametrically up to at most two constants) in all three models. Except for retirement flexibility and borrowing constraints these models share common features: irreversible retirement,[2] a constant mortality rate, different marginal utility per unit of consumption before and after retirement, possibly stochastic labor income, bequest, and actuarially fair life insurance.[3]

The first model serves as a benchmark, it has an exogenous mandatory retirement date and allows limited borrowing.[4] The second model considers voluntary retirement and also allows limited borrowing. This model seems intractable in the primal, so we solve it in the dual (i.e., as a function of the marginal utility of wealth) and obtain an explicit parametric solution up to a constant that is easy to determine numerically. We show that there exists a critical wealth-to-wage ratio above which it is optimal to retire. In addition, if labor income does not have a highly positive market exposure, human capital (the present value of future labor income) has a negative beta with any efficient portfolio ("the market"). This is because if the wage is nearly constant, it

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[1] The U.S. News and World Report (October 2004, "Preserving your portfolio," p. 66) quotes a pension consultant of Hewitt Associates who says that investors have a strange tendency to be "overly aggressive until the day they retire. Then they become overly conservative. It's like a light switch."

[2] Irreversibility is a stark assumption that emphasizes the fact that an employee's value is much less working part-time than full-time (see, e.g., Gustman and Steinmeier [7]). Also, the marketability of an employee declines rapidly while not working. Many variations are possible; for example, we have solved a model allowing part-time employment at a lower wage after retirement. The formulation in this paper has the merit of producing clean results that are easy to interpret.

[3] In general, many age-dependent factors may induce retirement, including declining health, reduced productivity, short remaining life expectancy, and institutions such as public and private pension plans. In this paper, we abstract from explicit age-dependent factors and focus on the pure impact of wealth and portfolio considerations.

[4] See Lazear [15] for why mandatory retirement may be optimal. An alternative model of mandatory retirement that allows for early retirement is more complicated because of the extra time dimension, but can be solved using the randomization method employed by Liu and Loewenstein [16] (see also Panageas and Farhi [19]). Our simpler assumption is a better benchmark because we can solve the model exactly and it is easier to compare with the other models.