How should Central Banks determine and control their bank note inventory?

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Abstract

This paper looks at a relatively unresearched but important area in money and banking – namely the provision of currency by the Central Bank. One of the most important functions of Central Banking is the provision of liquidity to the economy. However, in fulfilling this function, Central Banks have to be prepared for unexpected money demand shocks as well as production, transportation and cost of capital constraints. The paper develops a dynamic cost minimizing note inventory model that solves for the Central Bank's optimal note order size and frequency. As part of the modeling exercise a value at risk model is used to solve for an inventory "cushion."
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1. Introduction

One of the most important functions undertaken by Central Banks is the provision of liquidity (or currency) to the economy. Shortage of currency supply might impose severe constraints on the real economy inducing an economic contraction. An oversupply, on the other hand, may have an inflationary effect. Recent experiences of under- or over-supply have been evident in many developed and emerging market countries.[1] Moreover, Sargent and Velde (2002) document numerous historic episodes, of considerable economic impact, when the mix of denominations in the currency supply has been disrupted.

Despite the importance of note (currency) supply (and its composition) to the economy the question of a Central Bank's optimal control and inventory management of its note supply has gone largely unresearched. Indeed, a review of the literature suggests just a few papers that have either directly or indirectly addressed this topic. Baumol (1952) proposed a simple static bank note inventory model in a rational expectations economy and more recently Ladany (1997) has proposed a simple discrete dynamic programming model for the new note ordering policy for the Bank of Israel. While there are also some papers indirectly related to the question of bank note inventory and control, e.g., Boeschoten and Fase (1992) on bank note demand forecasts and Berger et al. (1996) on payment system risks, the literature in this area is sparse.

While from the social welfare point of view the question of provision of notes to the economy is important, also of importance is the cost to the Central Bank of this provision or service. Indeed, an optimal note supply/inventory policy by a Central Bank would seek to meet the expected and unexpected demands for currency by the public/financial institutions at *minimum* cost. The focus on minimizing the cost was incorporated in the 1980 DIDMCA in the US. In general the Act requires the Federal Reserve Banks, when possible, to assess all operating costs as well as the imputed costs of capital and taxes that would be incurred by a profit-making firm. Since 2002, the Federal Reserve has made fundamental changes to the calculations used to set the imputed costs, Lopez et al. (2003). An important feature of the model developed in this paper is the introduction of a positive ''cost of capital'' for a Central Bank as well as note production and transportation costs. In this paper a dynamic programming model of inventory control is developed that fully takes into account the demand (both expected and unexpected) for bank notes as well as a number of real world constraints by assuming the Central Bank wishes to minimize its overall cost of note provision subject to meeting the economy's liquidity needs. The model provides the periodic optimal note order quantities, note order frequencies and inventory levels that minimize the Central Bank's costs of note provision subject to supply/demand for note constraints. An innovative feature of the model is the development of different approaches to determine an inventory ''cushion'' or reserves to account for the different inherited risks in banknote inventory manage-

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[1] For example, an undersupply may also lead to hoarding of certain notes and coins. Recent examples include India, and Zimbabwe. See for example, ENS Bureau, Indian Express Newspapers, August 15, 1997 and Post and courier, Zimbabwe, Harare, July 31, 2003.