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On the regularity of equilibria in dynamic economies

Juan Manuel Licari*

Department of Economics, University of Pennsylvania, Philadelphia, PA 19104, USA

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Abstract

For discrete-time, infinite-horizon economies that are represented by a sequence of interrelated smooth equations, we provide sufficient conditions to guarantee regularity of equilibria on a full measure set of economies. Our approach exploits the sequential nature of a dynamic economy. An equilibrium is represented by a sequence of endogenous variables that solves a sequence of systems of smooth equations. At every point in time, standard smooth analysis techniques are used. The task then is to link periods, in order to obtain properties of equilibria. This link is represented by a correspondence, whose parallel in the macro/capital theoretic literature is the law of motion for state variables. Regularity conditions for the systems of equations and continuity properties of the law of motion drive most of the results. The set of tools proposed in this paper can also be used to analyze other properties of dynamic macroeconomic models, in particular, robustness of equilibria to smooth perturbations of utility and production functions.

JEL classification: C61; D50; E00

Keywords: Equilibrium correspondence; Law of motion; Policy correspondence; Regular economies

1. Introduction

We analyze regularity properties of infinite-horizon, discrete-time economies whose equilibria can be represented as solutions to a sequence of interrelated smooth equations. Standard models in macroeconomics and capital theory usually have a sequential representation[1]. The study of

* Tel.: +1 215 735 2837.
E-mail address: licari@econ.upenn.edu (J.M. Licari).

[1] A large class of such models consists of economies where the agents face an intertemporal problem that can be solved recursively, using dynamic programming tools (e.g., Stokey et al., 1989).

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regularity (or more generally, determinacy) in dynamic economies has a long tradition in the economic literature. Its importance relies on the fact that qualitative analysis of equilibria usually depend on smooth techniques. Araujo and Scheinkman (1977) connected the smoothness of the solutions to single intertemporal optimization problems with the turnpike property. Santos (1991) studied regularity of the policy function for economies where dynamic programming tools can be applied. Both analyses are limited to economies where equilibria are represented as solutions to a single agent's optimization problem. Similarly, determinacy properties for infinite-dimensional models in equilibrium theory rely on the use of the so-called Negishi equations (see for example, Balasko, 1997; Shannon, 1999). For these approaches to be valid, the standard linear version of social welfare optimization has to hold at equilibrium. The main advantage of our results is that they can be applied to models with a sequential representation, but violation of the welfare theorems.

In our model, equilibria are defined by sequences of endogenous variables that solve a sequence of systems of smooth equations. An economy is represented by a vector of initial conditions and time-invariant parameters. We say that an economy is regular if the associated equilibria (if any) satisfy a regularity (rank) condition at every period. The parallel of this definition in macro/capital theory is the smoothness of the policy function. We provide sufficient conditions to guarantee regularity of equilibria on a full measure set of economies. To get this result, we follow two steps. First, we make use of standard smooth analysis techniques (following the Balasko program, due to Balasko (1988)) at every point in time. Second, we link subsequent periods with a correspondence, which plays the role of the law of motion for state variables in macro/capital theory. Regularity conditions for the systems of equations at every period and continuity properties of the law of motion give us regularity of equilibria. We also find that, for a given economy, at any point in time, the endogenous variable associated with that period can only take a finite number of values. In terms of the law of motion correspondence, it will consist of a finite collection of implicit smooth functions. In other words, the model evolves smoothly from period to period, with a finite number of possible branches.

An important contribution of the paper is that it provides new tools for analyzing dynamic models which can be used to explore properties beyond regularity. The techniques introduced here will be used for studying robustness of equilibria for macroeconomic models in a subsequent paper. The fact that we have regular equilibria allows us to analyze smooth perturbations of all the fundamentals of the economy. We are then able to check whether those results found for particular choices of utility and production functions hold if we (smoothly) perturb preferences and technology.

The rest of the paper is organized as follows. Section 2 deals with preliminaries and notation. We define equilibria, the law of motion, and the policy correspondence. In Section 3 we present the results. The use of smooth analysis techniques at every period is presented in Section 3.1. The main results are collected in Section 3.2, under Theorem 3.1. Two extensions are also considered: finiteness of the set of regular equilibria and openness of the full measure set of regular economies. Appendix A contains those definitions and theorems required for the paper to be self-contained.

2. Preliminaries and notation

Time is discrete and denoted by $t \in \mathbb{N}_0 \equiv \mathbb{N} \cup \{0\}$. For any $t \in \mathbb{N}_0$, consider the open sets $Y_t \subseteq \mathbb{R}^{s_t}$, $X_t \subseteq \mathbb{R}^{n_t}$, and $\Lambda \subseteq \mathbb{R}^m$, with typical elements y_t, x_t, and λ, respectively. Define the sets $\Xi_t \equiv (Y_t \times X_{t+1})$ and $\Theta_t \equiv (X_t \times \Lambda)$, with typical elements ξ_t and θ_t, and the C^1 mapping $\Phi_t : (\Xi_t \times \Theta_t) \to \mathbb{R}^{(s_t+n_{t+1})}$ s.t. $(\xi_t, \theta_t) \mapsto \Phi_t(\xi_t, \theta_t)$.