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Warranty Cost Analysis for Second-Hand Products

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Abstract—For second-hand products sold with warranty, the expected warranty cost for an item to the manufacturer, depends on

(i) the age and/or usage as well as the maintenance history for the item and
(ii) the terms of the warranty policy.

The paper develops probabilistic models to compute the expected warranty cost to the manufacturer when the items are sold with free replacement or *pro rata* warranties.

Keywords—Used product, FRW policy, Expected warranty cost.

1. INTRODUCTION

New products are appearing on the market at an ever increasing pace and improvements in technologies have resulted in higher product life. As a result, the sale of new products is often tied to a trade-in, resulting in a market for second-hand products. The market for second-hand products as a fraction of the total market (new + second-hand) has been continuously increasing. In the U.S.A. car market, the trade for second-hand cars was 40% of that for new cars in numbers and 22% in terms of dollar sales [1].

At the same time, customers are becoming more demanding in terms of protection against products which fail to perform satisfactorily. Most new products are sold with warranty. For a given product, the warranty cost (in a statistical sense) is the same for all items if the manufacturer has good quality control. In contrast, the warranty cost for each second-hand item differs due to age, usage, and maintenance history. Dealers of second-hand products need to estimate this cost and build it into the price structure. Failure to do so can result in the dealers incurring loss, as opposed to profit, with the sale of products. This issue is becoming more important in the context of many second-hand products, such as cars, where laws mandate that the dealer offer warranty and that there be mechanisms to ensure that the dealer can effectively service warranty claims. This paper deals with stochastic models to estimate the expected warranty cost for second-hand products sold with free replacement or *pro rata* warranty policies. We commence with a brief discussion of warranty cost analysis for new products in Section 2. Following this, we discuss the modelling of claims for second-hand products in Section 3, where we discuss two approaches to modelling. Sections 4 and 5 deal with expected warranty costs based on these two approaches for the free replacement warranty policy. Section 6 deals with expected warranty costs for the *pro rata* warranty policy. We conclude with a brief discussion of topics for future research in the final section.

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2. EXPECTED WARRANTY COSTS FOR NEW PRODUCTS

Warranty cost analysis for new products has received a lot of attention. The warranty costs depend on the type of product (consumer durable, industrial, or custom built items such as airplanes or tanks for the government) and the type of warranty. Blischke and Murthy [2] propose a taxonomy for warranty policies and discuss the different types of warranty policies that have been studied.

The cost analysis can be looked at from two different perspectives—manufacturer (or seller) and buyer. The modelling of claims over the warranty period is dependent on many factors. These include product reliability, usage of the product, and the rectification actions of the manufacturer. A comprehensive framework which looks at these and other issues in the context of warranties is discussed in [3]. A review of mathematical models for cost analysis of different warranty policies can be found in [4]. More details of the model formulations and cost analysis can be found in [5]. Discussion of other issues (for example, legal, marketing, accounting) can be found in [6].

3. MODELLING WARRANTY CLAIMS FOR SECOND-HAND PRODUCTS

Most products (for example, cars) can be viewed as systems comprising of several components. Whenever a component fails, it affects the performance of the system and results in a claim if the product is still covered under warranty. These failures occur in an uncertain manner and can be characterised in terms of the reliability of the components.

One can use two approaches to modelling warranty claims. In the first approach (Approach 1), the system is viewed as a black-box and the claims modelled as a point process with a specified intensity function $\Lambda(t)$ with t representing the age of the system. The function $\Lambda(t)$ is an increasing function of t indicating that the number of claims (in a statistical sense) increases with age. This type of characterisation is appropriate when system failures occur due to a component failure and the system can be made operational through repair or replacement of the failed component. Since the failed component is a small element of the system, the rectification action can be viewed as having negligible impact, and hence, the system failure intensity after repair being the same as that before failure. In the reliability literature, this is known as "minimal repair" (see, [7]).

In the second approach (Approach 2), the claims are characterised through failures at component level. Hence, here one models the expected warranty cost for each component and the total expected warranty cost for the system is given by the sum of the costs for the different components. When the component failures are independent, the costs associated with each component can be calculated separately. When the failures are dependent, the analysis becomes more complex. In this paper, we confine our attention to the case where the component failures are independent.

As mentioned earlier, the warranty cost for a used item depends on its age and past usage and maintenance history. In this paper, we look at models which ignore the past usage and maintenance history. As a result, the significant variables which affect the warranty cost are

(i) the age of the item (A) and the ages of the components (A_i), $1 \leq i \leq n$, at the time of sale,
(ii) the warranty policy, and
(iii) the rectification strategy used by the dealer.

Often the dealer has knowledge of the age (A) of the item. This is usually obtained from sources such as registration forms and/or log books. When one models at component level, the dealer might or might not know the true age A_i of component i. In this case, one needs to model this uncertainty. We shall discuss this further in Section 5.

We consider the following two types of warranty policies.