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# Random matrix theory and fund of funds portfolio optimisation

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**Abstract**

The proprietary nature of Hedge Fund investing means that it is common practise for managers to release minimal information about their returns. The construction of a fund of hedge funds portfolio requires a correlation matrix which often has to be estimated using a relatively small sample of monthly returns data which induces noise. In this paper, random matrix theory (RMT) is applied to a cross-correlation matrix $C$, constructed using hedge fund returns data. The analysis reveals a number of eigenvalues that deviate from the spectrum suggested by RMT. The components of the deviating eigenvectors are found to correspond to distinct *groups of strategies* that are applied by hedge fund managers. The inverse participation ratio is used to quantify the number of components that participate in each eigenvector. Finally, the correlation matrix is cleaned by separating the noisy part from the non-noisy part of $C$. This technique is found to greatly reduce the difference between the predicted and realised risk of a portfolio, leading to an improved risk profile for a fund of hedge funds.


*Keywords:* Random matrix theory; Hedge funds; Fund of funds; Correlation matrix; Portfolio optimisation

## 1. Introduction

A hedge fund is a lightly regulated private investment vehicle that may utilise a wide range of investment strategies and instruments. These funds may use short positions, derivatives, leverage and charge incentive-based fees. Normally, they are structured as limited partnerships or offshore investment companies. Hedge funds pursue positive returns in all markets and hence are described as "absolute return" strategies.

Hedge funds are utilised by pension funds, high net-worth individuals and institutions, due to their low correlation to traditional long-only investment strategies. The incentive-based performance fees, earned by hedge fund managers, align the interest of the hedge fund manager with that of the investor. The performance of hedge funds has been impressive, with the various hedge fund indices providing higher returns, with lower volatility, than traditional assets over many years. As of the end of the first quarter 2006 the total assets managed by hedge funds world wide is estimated at $1.25 trillion [1]. Hedge funds generally only report their

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returns on a monthly basis and this means that there is a very limited amount of data available to study as databases of hedge fund returns have only been in operation for about 15 years. This is in keeping with the highly secretive, proprietary nature of hedge fund investing. The amount of information reported by a hedge fund about how and where it is producing its returns is often limited to sectoral overviews and strategy allocations. For an introduction to hedge funds see Refs. [2,3].

Significant diversification benefits can be gained by investing in a variety of hedge fund strategies, due to the presence of low and even negative correlations between different hedge fund strategies. Such strategies can be broken up into two general categories: directional and market neutral. Directional strategies, (for example long/short equity, emerging markets, macro and managed futures) have a high risk, high return profile and act as return enhancers to a traditional portfolio. Market neutral strategies, (for example convertible arbitrage, equity market neutral and fixed income arbitrage) deploy a low risk profile and act as a substitute for some proportion of the fixed income holdings in an investors portfolio [2,3].

A fund of hedge funds allows investors to have access to a large diverse portfolio of hedge funds without having to carry out due diligence on each individual manager. The diversification benefits provided by fund of funds are brought about by investing in a number of funds that have a low correlation to each other. These correlations are often calculated by using equally weighted fund returns and can contain a significant amount of noise due to the very small amount of returns data available for hedge funds [3].

In this paper we apply random matrix theory (RMT) to hedge fund returns data with the aim of reducing the levels of noise in these correlation matrices formed from this data and hence constructing a fund of hedge funds with an improved risk profile. Previous studies have used the information found in the RMT defined deviating eigenvalues of a correlation matrix as inputs into a minimum spanning tree [4] to enable characterisation of hedge fund strategies. In this paper, the components of the deviating eigenvectors are shown to correspond to distinct groups of strategies that are applied by hedge fund managers and this is exploited to construct a portfolio with reduced levels of risk.

This paper is organised as follows: in Section 2 we review RMT and discuss its use in the extraction of information from a correlation matrix of hedge fund returns using RMT techniques. In Section 3, we look at the results obtained applying RMT to hedge funds and, in the final section, we draw our conclusions.

## 2. Methods

### 2.1. Random matrix theory

Given returns $G_i(t)$, $i = 1, \ldots, N$, of a collection of hedge funds we define a normalised return in order to standardise the different fund volatilities. We normalise $G_i$ with respect to its standard deviation $\sigma_i$ as follows:

$$g_i(t) = \frac{G_i(t) - \widehat{G_i(t)}}{\sigma_i}, \tag{1}$$

where $\sigma_i$ is the standard deviation of $G_i$ for assets $i = 1, \ldots, N$ and $\widehat{G_i}$ is the time average of $G_i$ over the period studied.

Then the equal time cross-correlation matrix is expressed in terms of $g_i(t)$

$$C_{ij} \equiv \langle g_i(t) g_j(t) \rangle. \tag{2}$$

The elements of $C_{ij}$ are limited to the domain $-1 \leqslant C_{ij} \leqslant 1$, where $C_{ij} = 1$ defines perfect correlation between funds, $C_{ij} = -1$ corresponds to perfect anti-correlation and $C_{ij} = 0$ corresponds to uncorrelated funds. In matrix notation, the correlation matrix can be expressed as

$$\mathbf{C} = \frac{1}{T} \mathbf{G}\mathbf{G}^t, \tag{3}$$

where $\mathbf{G}$ is an $N \times T$ matrix with elements $g_{it}$.